# Approximations for the Feedback Vertex Set Problem

## CS 5114 Initial Project Proposal

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The Feedback Vertex Set (FVS) problem takes as an instance a graph $G = (V, E)$ and looks for a solution that is a set $F \subset V$ of minimum cardinality such that every cycle of $G$ contains at least one vertex of $F$. Viewed another way, the problem is to find the smallest number of vertices whose removal renders $G$ acyclic. The graph may be either directed or undirected. The vertices may also be weighted, in which case the "minimum cardinality" is replaced by "minimum total weight." All four variations of FVS are NP-complete, but an optimal solution can be approximated within a factor of 2 for unweighted or weighted undirected graphs, as shown by Bafna, Berman, and Fujito [1, 2]. Their approximation algorithm uses the local-ratio approach originally used by Bar-Yehuda and Even [3] to approximate an optimal solution for the vertex cover problem. Previously, Bar-Yehuda, et al. [4] established a factor of 4 approximation for FVS in the case of unweighted undirected graphs. Subsequently, Fujito [5] used the local-ratio approach for other vertex deletion problems. The vertex cover and FVS problems are closely related, as a polynomial-time factor-$r$ approximation algorithm for FVS can be converted into a polynomial-time factor-$r$ approximation algorithm for vertex cover. Since vertex cover has been much studied (more intensely than FVS) for years and the best constant approximation factor obtained is 2, reducing the constant factor for FVS below 2 is too difficult to contemplate in this proposal.

I propose to attack the approximation of FVS from two directions — theoretical and experimental. From a theoretical direction, I will follow the intuition that many disjoint small cycles require a like number of vertices in an FVS $F$. More precisely, suppose an algorithm greedily finds vertex-disjoint cycles $C_1, C_2, \ldots, C_k$ in $G$. Then any FVS $F$ for $G$ will contain at least one vertex from each $C_i$, making a total cardinality of at least $k$ for $F$. To select which vertex of each $C_i$ goes into $F$, I will examine three approaches. In the first approach, I will use the local-ratio approach of the Bafna, Berman, and Fujito algorithm as the tie-breaker. In the second approach, I will select the lightest vertex in the cycle, assuming that $G$ is a weighted graph. In the third approach, I will select a vertex in $C_i$ based on the number of not yet broken small cycles in $G$ that it breaks; there are several ways to pursue this approach depending on how small cycles are counted and on what order the vertices are selected. For all algorithms that I propose, I will analyze their approximation ratio and time complexity in the worst case and, if possible, in the average case.

From an experimental direction, I will implement some of my own algorithms above and the Bafna, Berman, and Fujito algorithm and compare them on random instances. The random instances will be generated using the standard random graph model with two parameters $n$ and $p$. A random $G = (V, E)$ with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ is generated by deciding whether each of the $\binom{n}{2}$ edges $(v_i, v_j)$ is in $E$ by an independent Bernoulli trial with probability $p$. I will generate at least 10 random graphs for each combination of $n$ and $p$ chosen over suitable ranges (perhaps, $50 \leq n \leq 1000$ and $p \in \{0.01, 0.1, 0.25, 0.5, 0.75, 0.9, 0.99\}$). Performance results will be tabulated and discussed.

After completing the theoretical and experimental analyses, I will recommend which FVS algorithm to utilize for particular kinds of instances. I will also report any open questions that remain to be pursued.

## References

[1] V. Bafna, P. Berman, and T. Fujito, *Constant ratio approximations of feedback vertex sets in weighted undirected graphs*, Tech. Rep. TR-96-29, DIMACS, Piscataway, NJ, 1996.

[2] ——, *A 2-approximation algorithm for the undirected feedback vertex set problem*, SIAM J. Discrete Math., 12 (1999), pp. 289–297 (electronic).

[3] R. Bar-Yehuda and S. Even, *A local-ratio theorem for approximating the weighted vertex cover problem*, in Analysis and Design of Algorithms for Combinatorial Problems (Udine, 1982), North-Holland, Amsterdam, 1985, pp. 27–45.

[4] R. Bar-Yehuda, D. Geiger, J. Naor, and R. M. Roth, *Approximation algorithms for the feedback vertex set problem with applications to constraint satisfaction and Bayesian inference*, SIAM J. Comput., 27 (1998), pp. 942–959 (electronic).

[5] T. Fujito, *A unified approximation algorithm for node-deletion problems*, Discrete Appl. Math., 86 (1998), pp. 213–231.