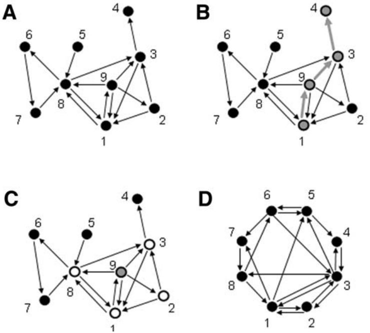
CS 6824: Small World of Brain Networks



T. M. Murali

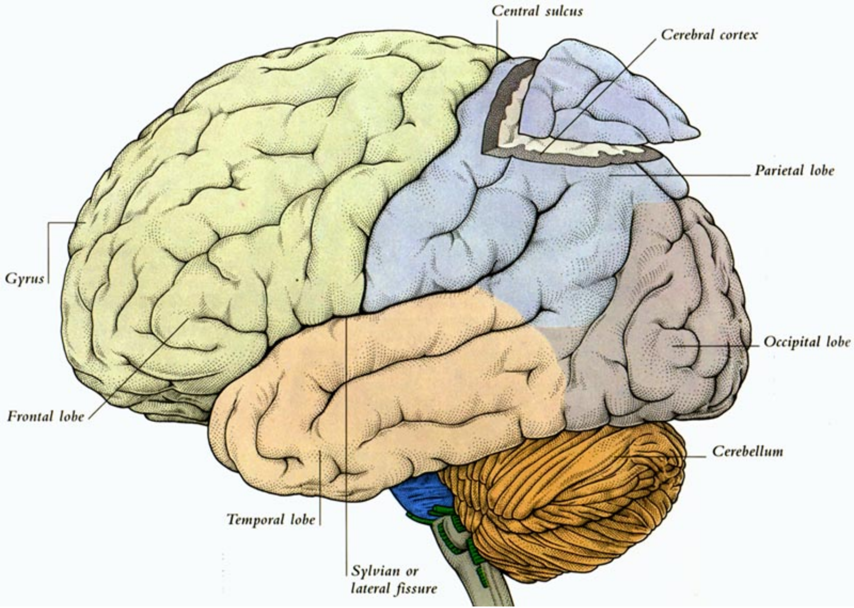
February 13, 2018

Motivation

- The Watts-Strogatz paper set off a storm of research.
- It has nearly 35,000 citations. Even in 2004, it had more than 2,100 citations.
- The *C. elegans* neuronal network is small-world.

Do mammalian brain networks have the small world property?

Visual and Cerebral Cortices



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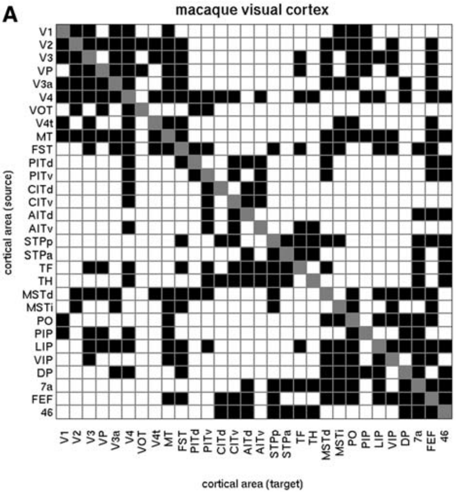
Datasets: Macaque Visual Cortex



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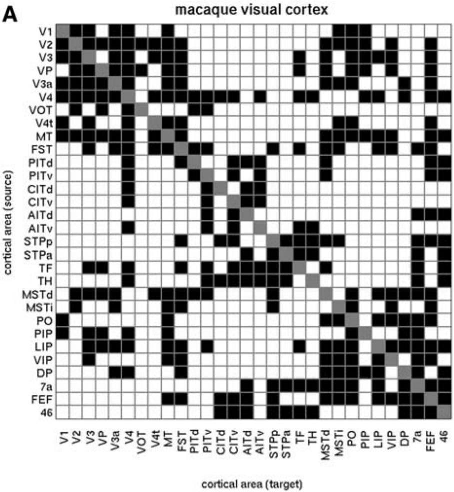

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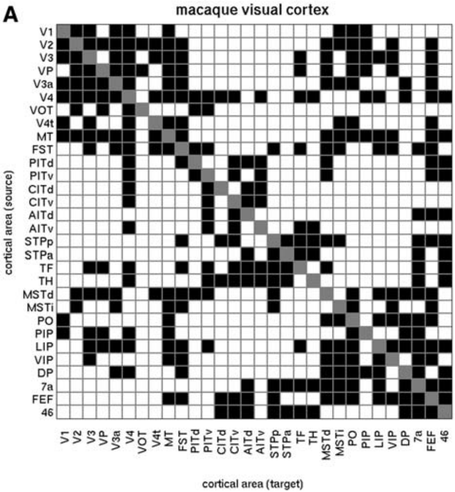

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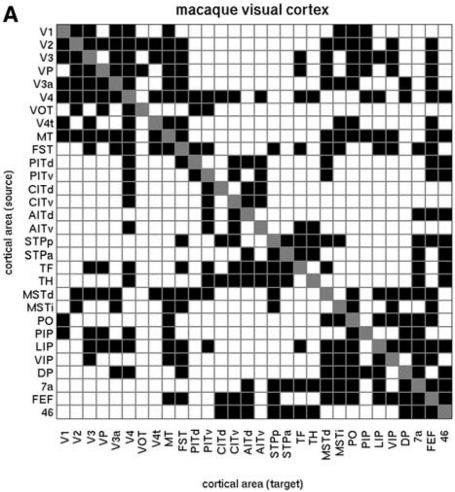

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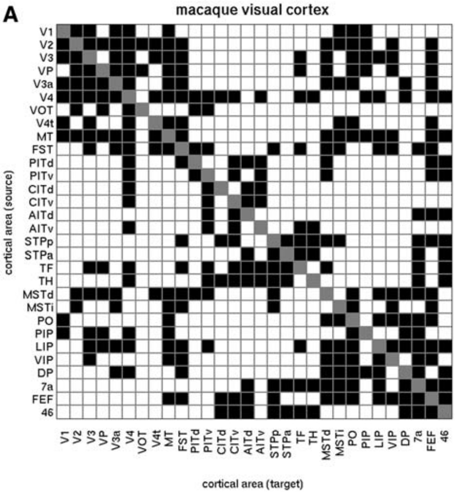

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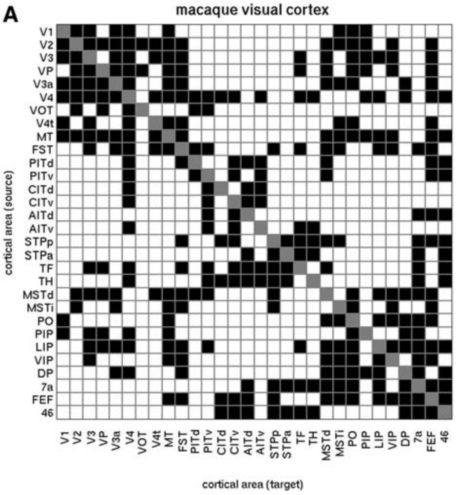

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- $n =$, $m =$. (The authors use N for the number of nodes and K for the number of edges.)

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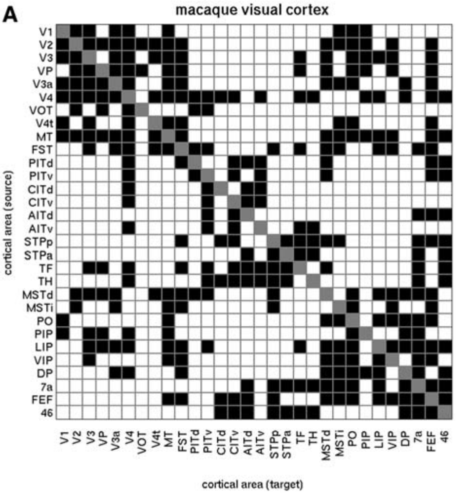

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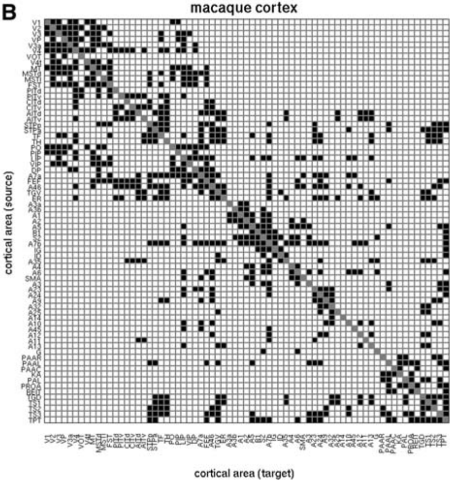

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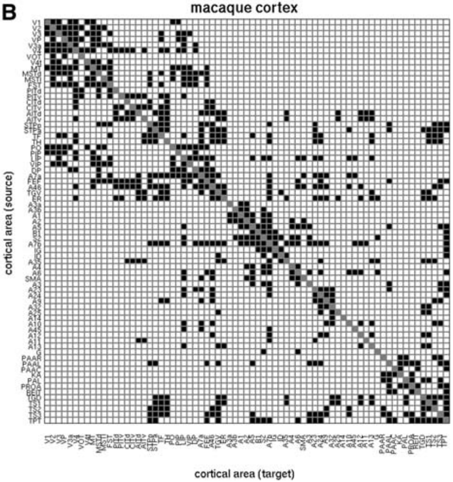

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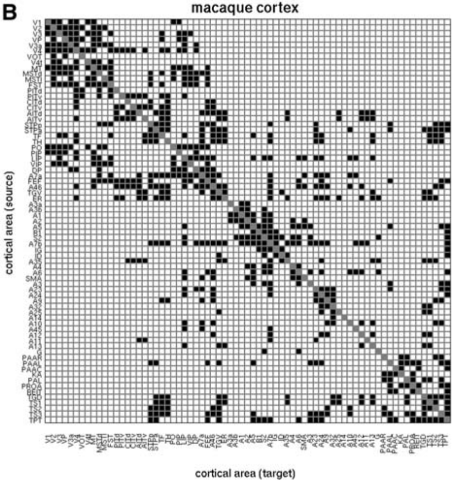

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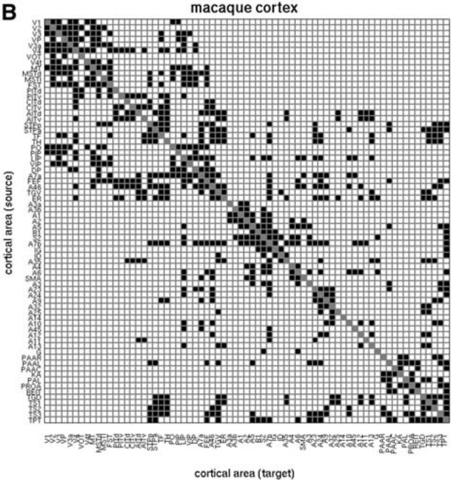

- $n = 71, m = 746$.
- What is the relation between this graph and the one for the macaque visual cortex? Most of the edges in the previous graph are in this one.

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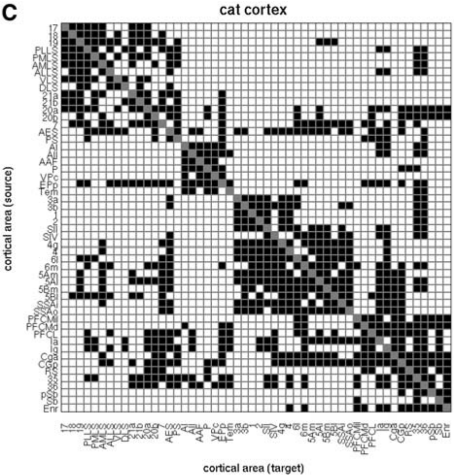

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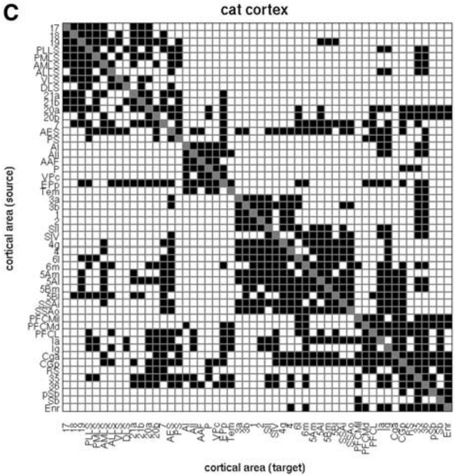

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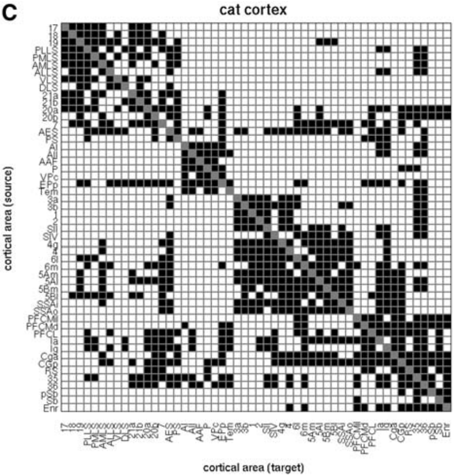

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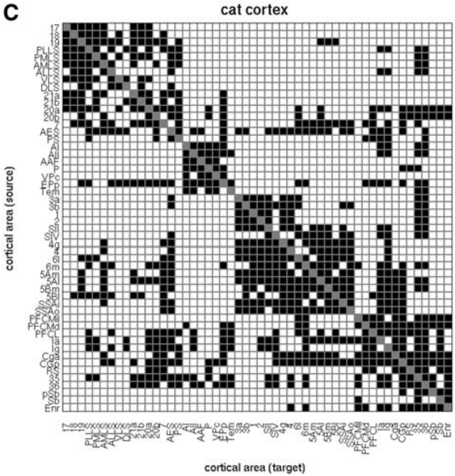

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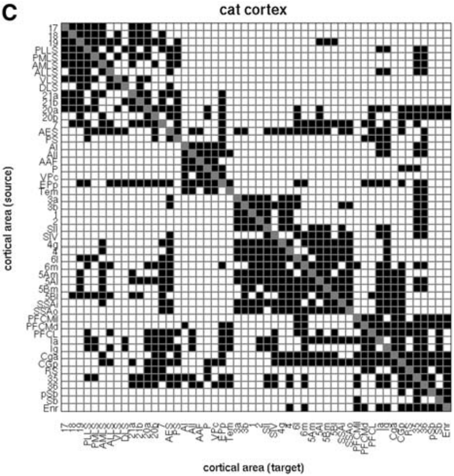

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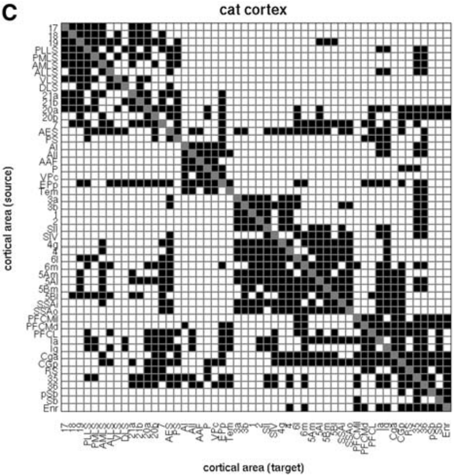

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- Are these datasets "large-scale"?

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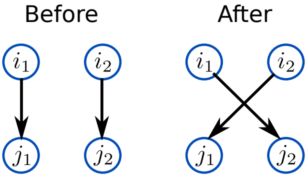

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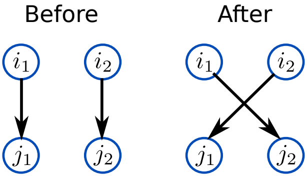

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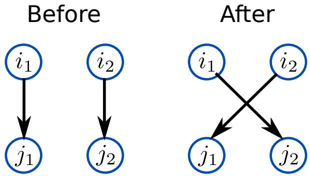

- Degree-preserving lattice matrix: Very poorly specified. Will need to read the code to understand.

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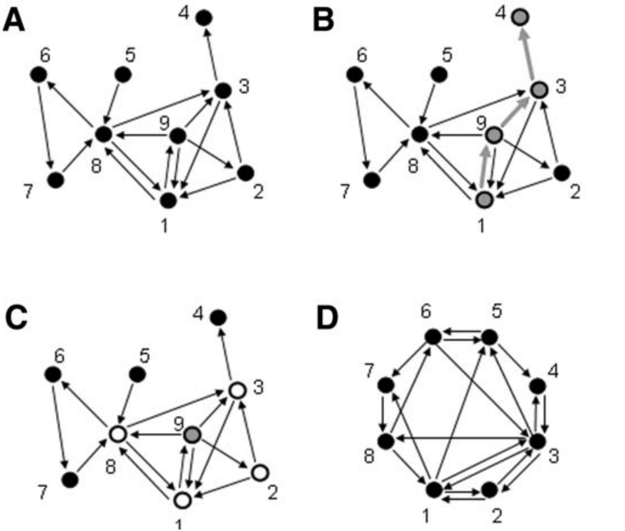
  - Clustering coefficient, $c(v)$, called "cluster index" and $\lambda(v)$ in this paper.

Computing $l(G)$ and $c(G)$ for Directed Graphs



- Appropriately generalise definitions for undirected graphs.

Scaling $l(G)$ and $c(G)$

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- How do we interpret these quantities? A small world network will have small $l_{\text{scl}}(G)$ and large $c_{\text{scl}}(G)$.

Results for $l(G)$ and $c(G)$

Table 1. Path Length (λ, λ_{scl}) and Cluster Index (γ, γ_{scl}) for Large-Scale Connection Matrices of Cortico-Cortical Pathways

Topology	λ	γ	λ_{scl}	γ_{scl}
MVC	1.7256	0.5313	0.2188	0.5645
$R_{30,311}$	1.6680 (0.0038)*	0.3616 (0.0048) *		
$L_{30,311}$	1.9313 (0.0018)*	0.6622 (0.0000) *		
$Rio_{30,311}$	1.6880 (0.0033)*	0.4305 (0.0059) *		
$Lio_{30,311}$	1.8190 (0.0391)	0.6214 (0.0243)		
MC	2.3769	0.4614	0.1927	0.6117
$R_{71,746}$	2.0310 (0.0051)*	0.1497 (0.0030) *		
$L_{71,746}$	3.8262 (0.0099)*	0.6593 (0.0002) *		
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$Lio_{71,746}$	2.8901 (0.1173)*	0.8992 (0.0211) *		
CC	1.8114	0.5514	0.2498	0.6292
$R_{52,820}$	1.7014 (0.0013)*	0.3103 (0.0026) *		
$L_{52,820}$	2.1418 (0.0024)*	0.6933 (0.0000) *		
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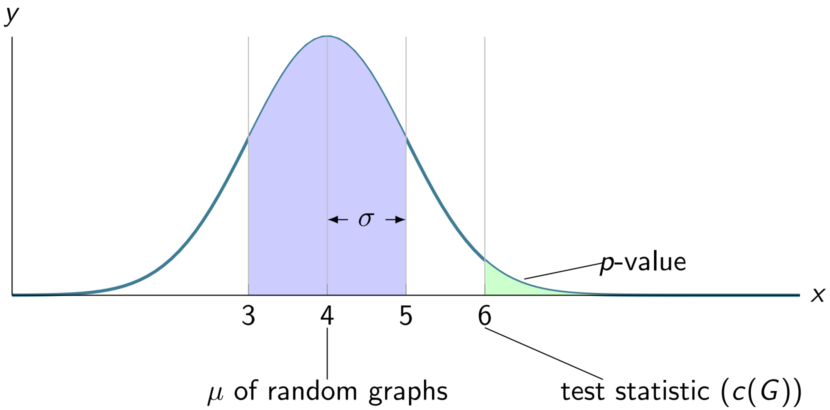
$Lio_{52,820}$	1.8570 (0.0283)	0.5893 (0.0172)		

Measures for reference cases represent means and standard deviations (in brackets) for 10 exemplars. Topologies: MVC = macaque visual cortex (Fig. 1A), MC = macaque cortex (Fig. 1B), CC = cat cortex (Fig. 1C), $R_{N,K}$ = random, $L_{N,K}$ = lattice, $Rio_{N,K}$, $Lio_{N,K}$ = random, lattice matrices with in-degree and out-degree distribution preserved. Statistical significance for all comparisons between cortical matrices and random, lattice, Rio, or Lio matrices marked by "*" are $p<0.001$, the remaining comparisons are $p<0.05$.

Statistical Significance



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$L_{71,746}$	3.8262 (0.0099)*	0.6593 (0.0002) *		
$Rio_{71,746}$	2.1159 (0.0133)*	0.2409 (0.0047) *		
$Lio_{71,746}$	2.8901 (0.1173)*	0.8992 (0.0211) *		
CC	1.8114	0.5514	0.2498	0.6292
$R_{52,820}$	1.7014 (0.0013)*	0.3103 (0.0026) *		
$L_{52,820}$	2.1418 (0.0024)*	0.6933 (0.0000) *		
$Rio_{52,820}$	1.7217 (0.0037)*	0.4023 (0.0030) *		
$Lio_{52,820}$	1.8570 (0.0283)	0.5893 (0.0172)		

< 0.5 > 0.5

Measures for reference cases represent means and standard deviations (in brackets) for 10 exemplars. Topologies: MVC = macaque visual cortex (Fig. 1A), MC = macaque cortex (Fig. 1B), CC = cat cortex (Fig. 1C), $R_{N,K}$ = random, $L_{N,K}$ = lattice, $Rio_{N,K}$, $Lio_{N,K}$ = random, lattice matrices with in-degree and out-degree distribution preserved. Statistical significance for all comparisons between cortical matrices and random, lattice, Rio, or Lio matrices marked by "*" are p<0.001, the remaining comparisons are p<0.05.

Results for $l(G)$ and $c(G)$

Table 1. Path Length (λ, λ_{scl}) and Cluster Index (γ, γ_{scl}) for Large-Scale Connection Matrices of Cortico-Cortical Pathways

Topology	λ	γ	λ_{scl}	γ_{scl}
MVC	1.7256	0.5313	0.2188	0.5645
$R_{30,311}$	1.6680 (0.0038)*	0.3616 (0.0048) *		
$L_{30,311}$	1.9313 (0.0018)*	0.6622 (0.0000) *		
$Rio_{30,311}$	1.6880 (0.0033)*	0.4305 (0.0059) *		
$Lio_{30,311}$	1.8190 (0.0391)	0.6214 (0.0243)		
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Observations on $l(G)$ and $c(G)$

- Values of $l(G)$ and $c(G)$ for connectomes are higher than for random networks with the same number of nodes and edges; difference is statistically significant.
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- Caveats:
  - *p*-values are likely to be underestimated. They should be estimated from empirical distributions built from many more random samples.
  - No indication of correction for testing multiple hypotheses.
  - No *p*-value associated with the scaled values of $l(G)$ and $c(G)$.

Computing Small-Worldness

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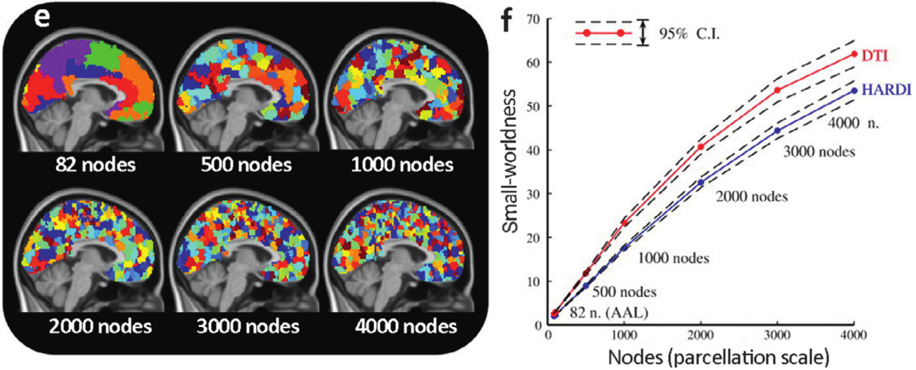
Parcellation Affects $\sigma(G)$



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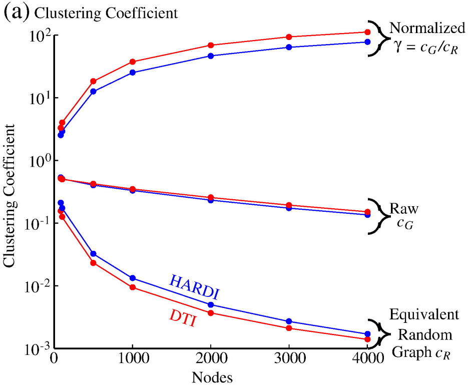

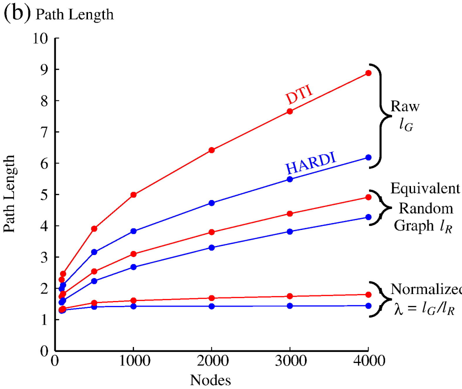

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- New tract-tracing techniques have yielded networks with a density of 0.66, with no small world features (Markov *et al.* 2013).
- These global measures obscure more meaningful variations occurring at the level of nodes or subgraphs.