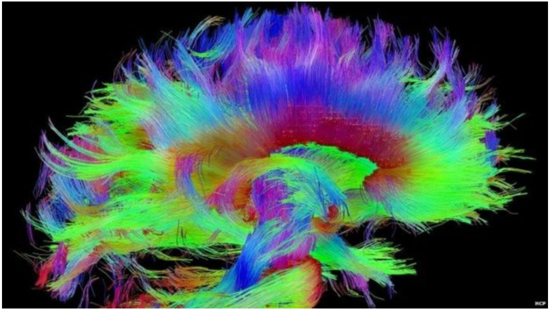
CS 4984: Connectivity Matrices and Node Degrees

T. M. Murali

February 6, 2018



Definition of an Undirected Graph

- *Weighted, undirected graph* $G = (V, E, w)$:
  - set V of nodes.
  - set E of edges.
    - ★ Each element of E is an unordered pair of nodes.
    - ★ Exactly one edge between any pair of nodes (G is not a multigraph).
    - ★ G contains no self loops, i.e., edges of the form (u, u).
  - Each edge (u, v) in E has a weight $w(u, v) \in \mathbb{R}$
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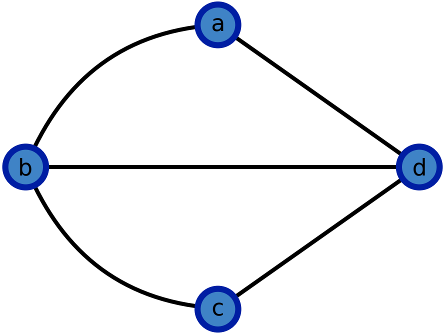

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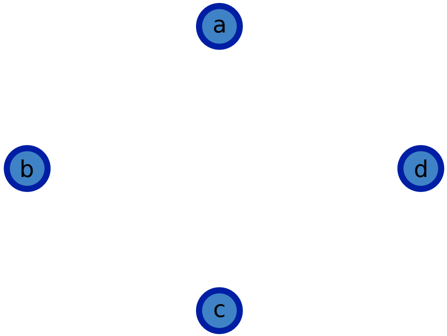

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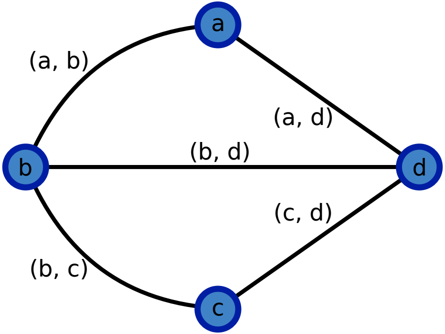

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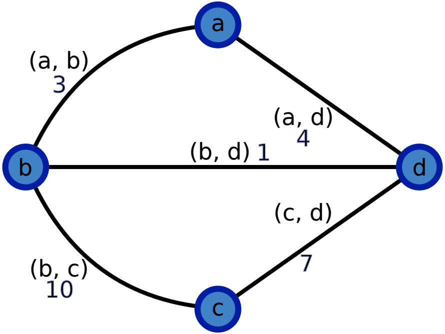

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Types of Brain Graphs

	Structural connectivity	Functional connectivity
Microscale		
Mesoscale		
Macroscale		

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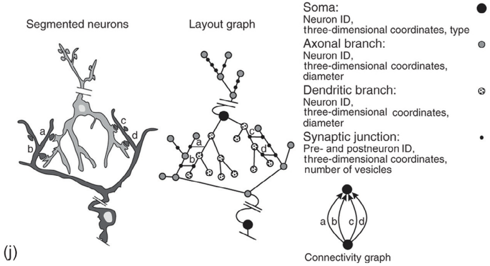

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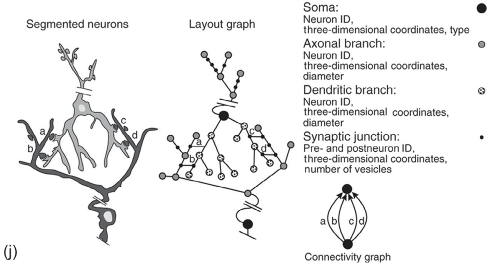

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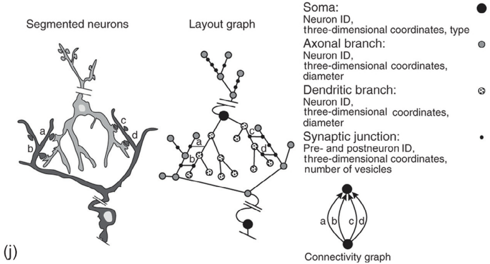

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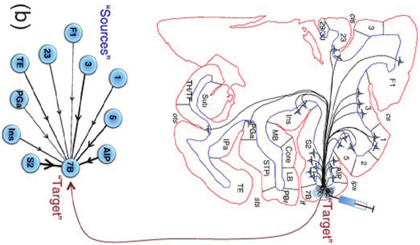

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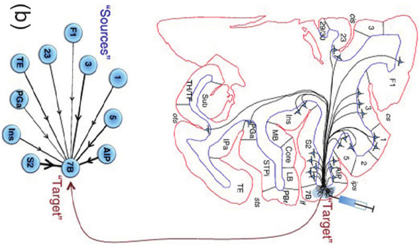

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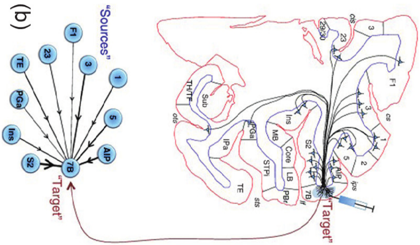

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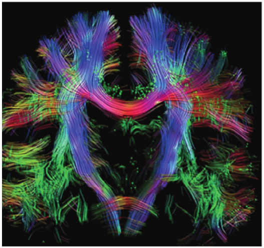

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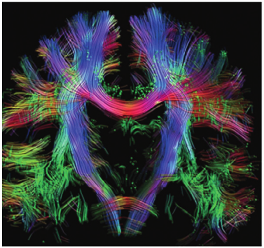

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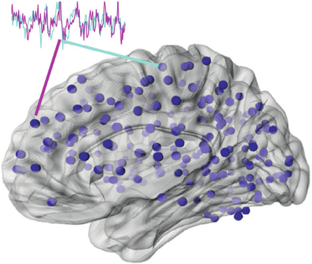

(c)

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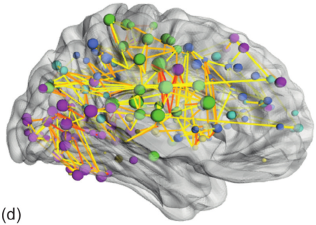
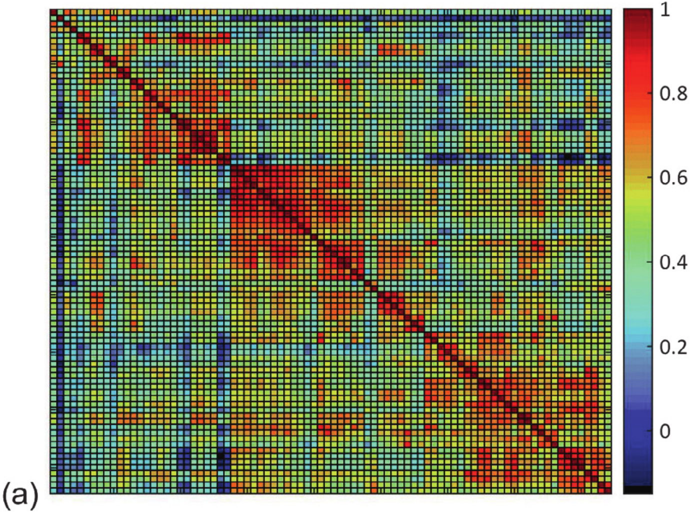


(d)

Thresholding and Binarisation



Human functional connectivity matrix from fMRI data.
Every element has a nonzero value.

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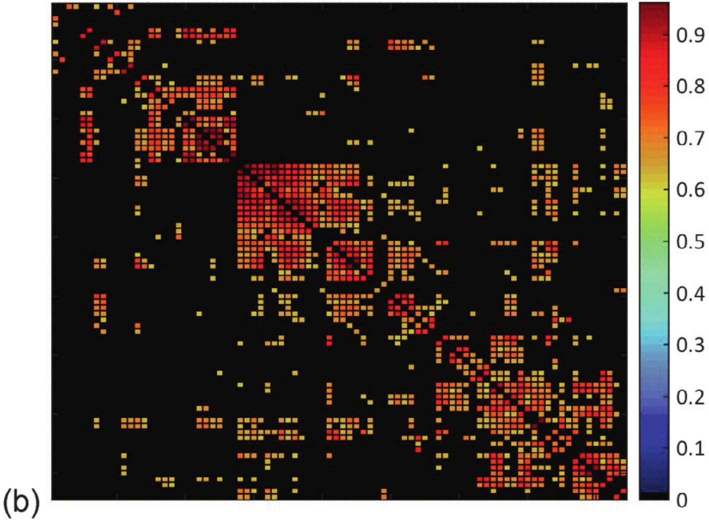

Matrix after thresholding to retain only the 20% strongest weights.

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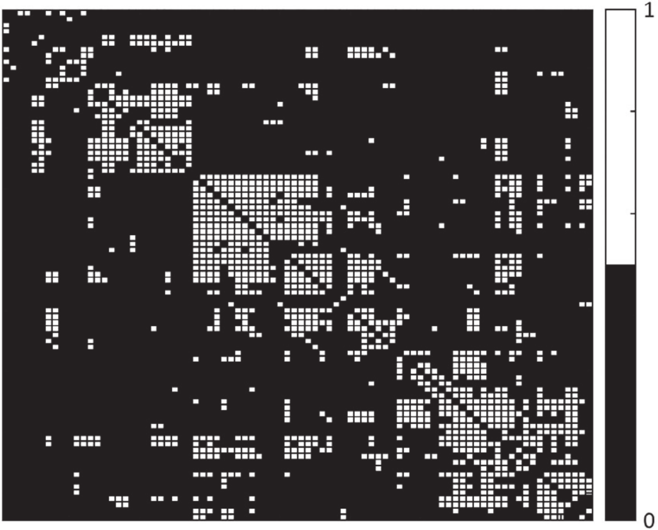

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Matrix after thresholding and binarisation.

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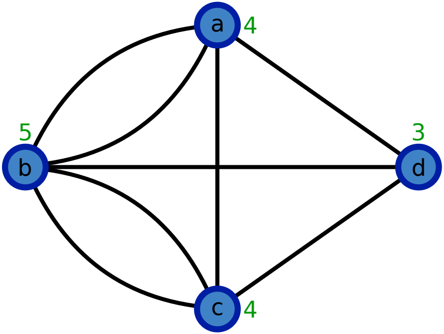
- We can modify these ideas for directed graphs.

Implementing Hierholzer's Algorithm



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while $d(u) > 0$ **do**
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end while

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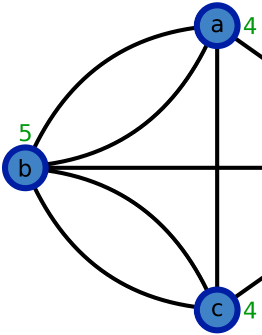

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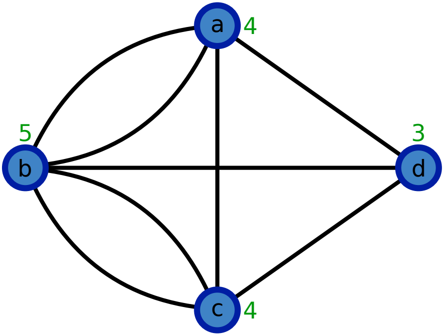

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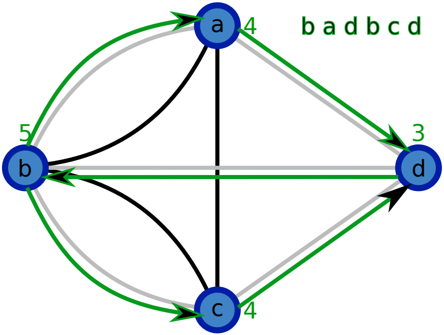

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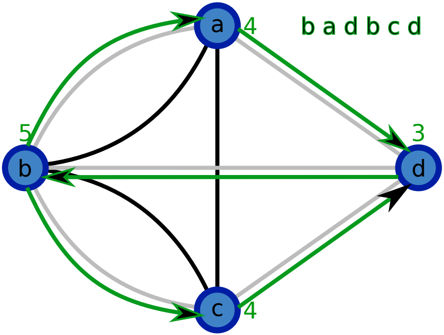

$u \leftarrow s$ $\#u$ is the current node.
while $d(u) > 0$ **do**
  Output u.
  Let v be a neighbour of u.
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end while

	Adjacency matrix	**Adjacency list**
Determine $d(u)$	Maintain array of node degrees. $O(1)$ time	Same idea
Find a neighbour v of u	Traverse row for u. $O(n)$ time.	v is first node in $\texttt{Adj}[u]$.
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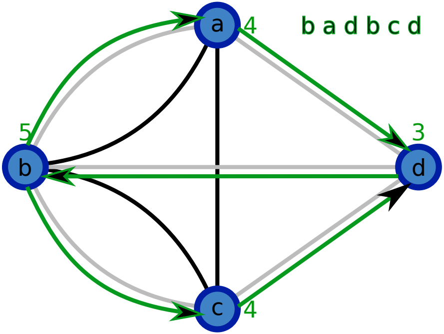

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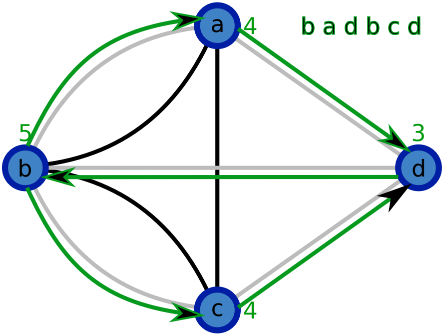

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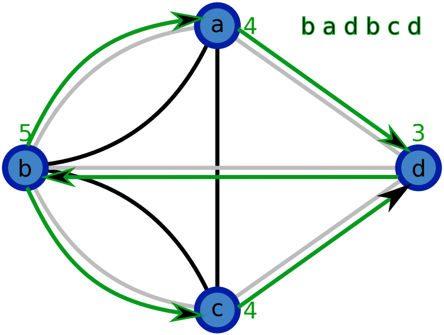

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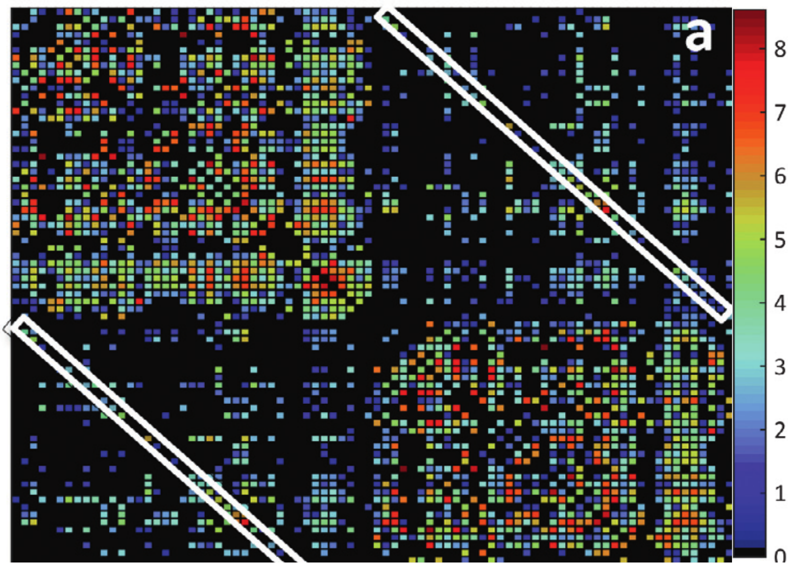

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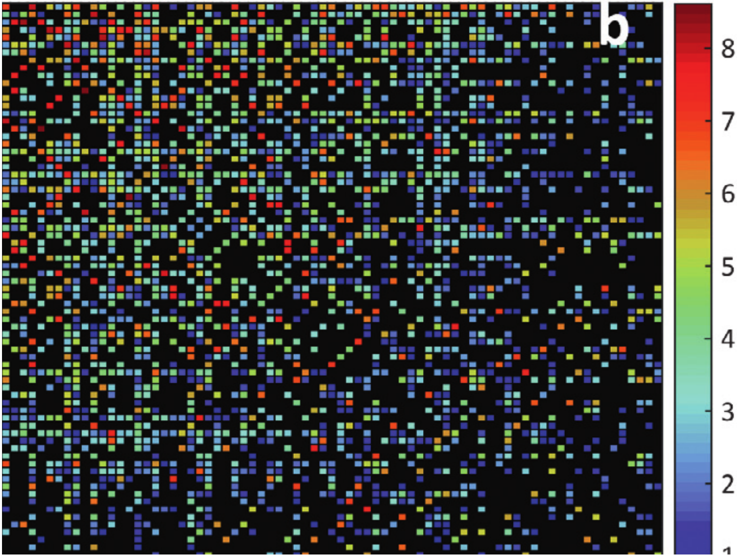

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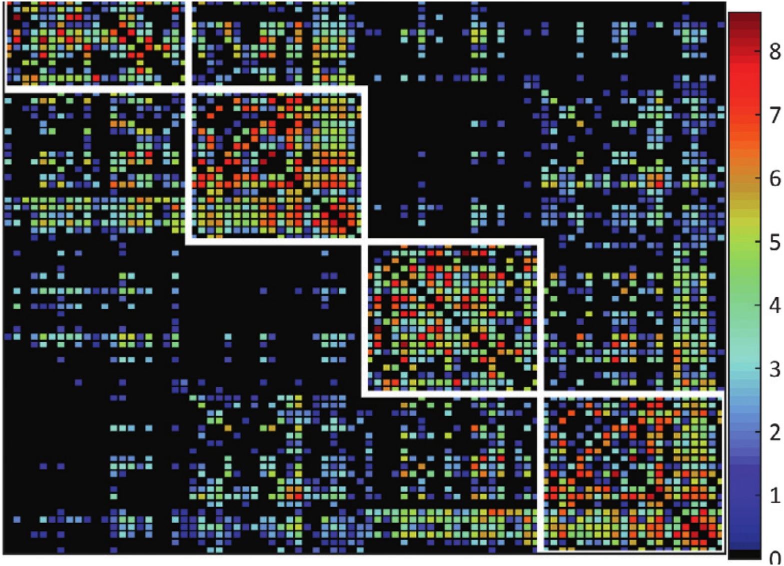
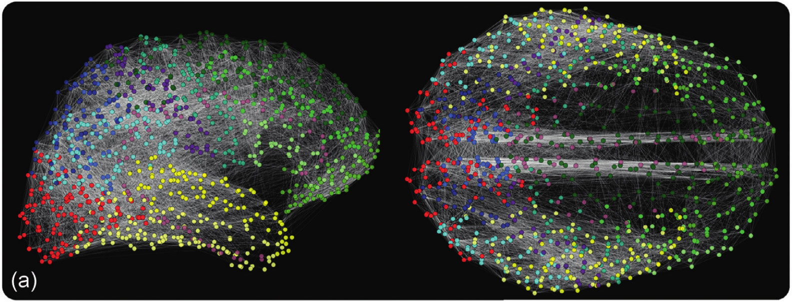
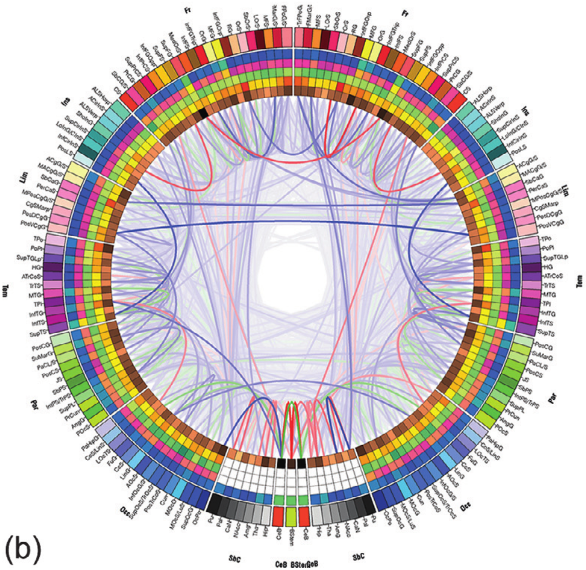


Anatomical Projection

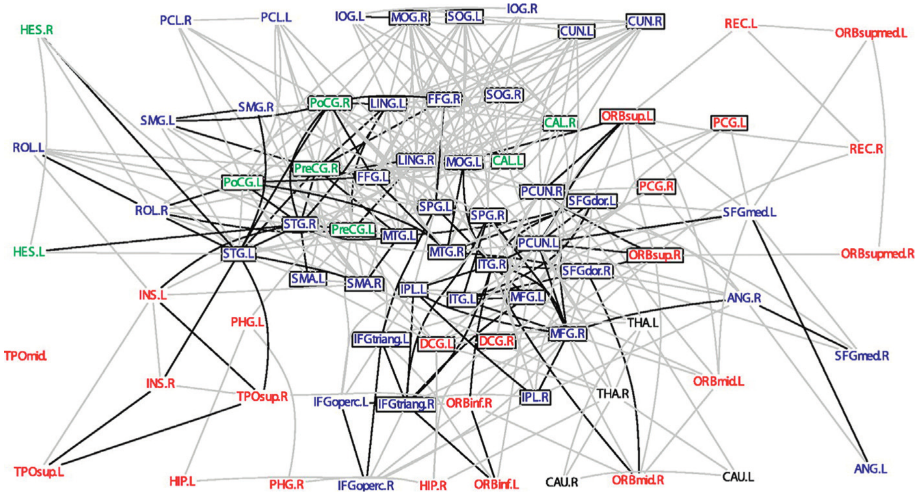


Circular Layout

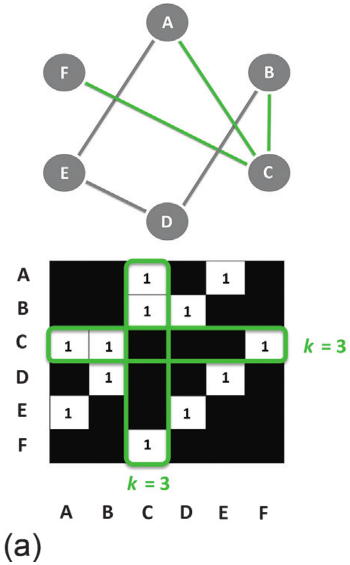


Force-Directed Layout

Spring-Embedded Layout



Node Degree



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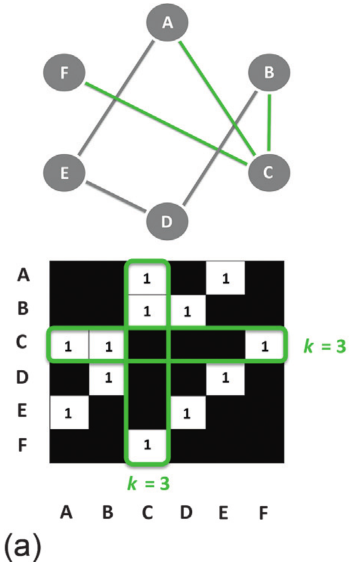

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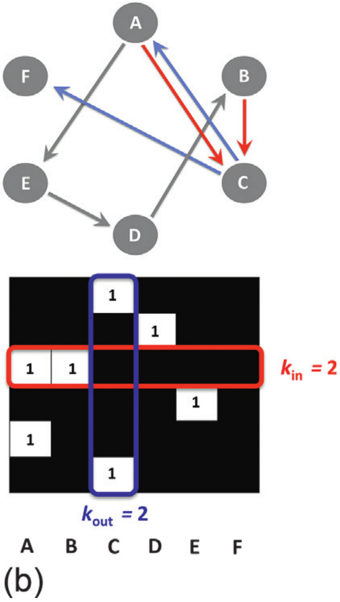

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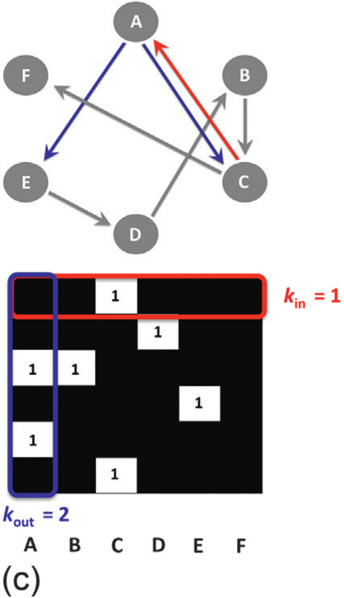

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$$\sum_{k \geq 0} kp(k) = \frac{1}{n} \sum_{k \geq 0} kn(k) = \frac{1}{n} \sum_{v \in V} d(v) = \frac{2m}{n}$$