More Realistic Adversarial Settings

Virginia Tech CS4804
Introduction to Artificial Intelligence

Outline

- Move ordering
- Stochastic games
- (Partially-observable games)


Max
Min
...

MINIMAX(*s*) =
    if TERMINAL-TEST(*s*) then UTILITY(*s*)
    if PLAYER(*s*) = MAX then
        max of MINIMAX(RESULT(*s*,*a*)) for *a* in ACTIONS(s)
    if PLAYER(s) = MIN then
        min of MINIMAX(RESULT(*s*,*a*)) for *a* in ACTIONS(s)

```
function MINIMAX-DECISION(state) returns an action
   return arg max_{a ∈ ACTIONS(s)} MIN-VALUE(RESULT(state, a))
```

```
function MAX-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for each a in ACTIONS(state) do
      v ← MAX(v, MIN-VALUE(RESULT(s, a)))
   return v
```

```
function MIN-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← ∞
   for each a in ACTIONS(state) do
      v ← MIN(v, MAX-VALUE(RESULT(s, a)))
   return v
```

```
function ALPHA-BETA-SEARCH(state) returns an action
   v ← MAX-VALUE(state, −∞, +∞)
   return the action in ACTIONS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for each a in ACTIONS(state) do
      v ← MAX(v, MIN-VALUE(RESULT(s,a), α, β))
      if v ≥ β then return v
      α ← MAX(α, v)
   return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← +∞
   for each a in ACTIONS(state) do
      v ← MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
      if v ≤ α then return v
      β ← MIN(β, v)
   return v
```

NATURE | NEWS

Game theorists crack poker

An 'essentially unbeatable' algorithm for the popular card game points to strategies for solving real-life problems without having complete information.

Philip Ball

08 January 2015

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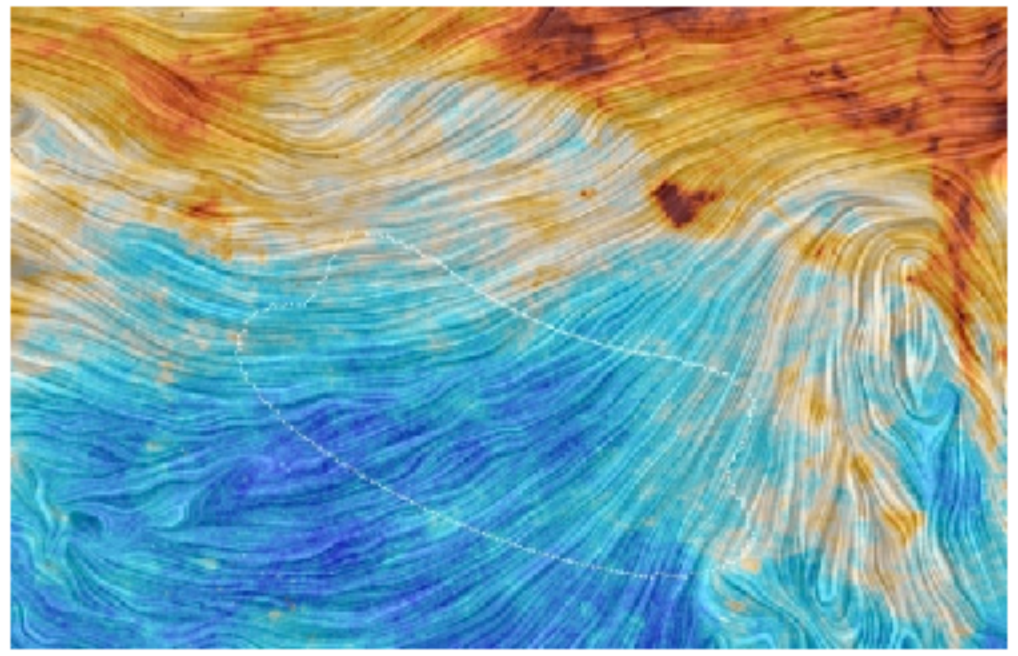

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Galactic dust confirmed as the source of the signal that researchers thought was evidence for gravitational waves from the early Universe.

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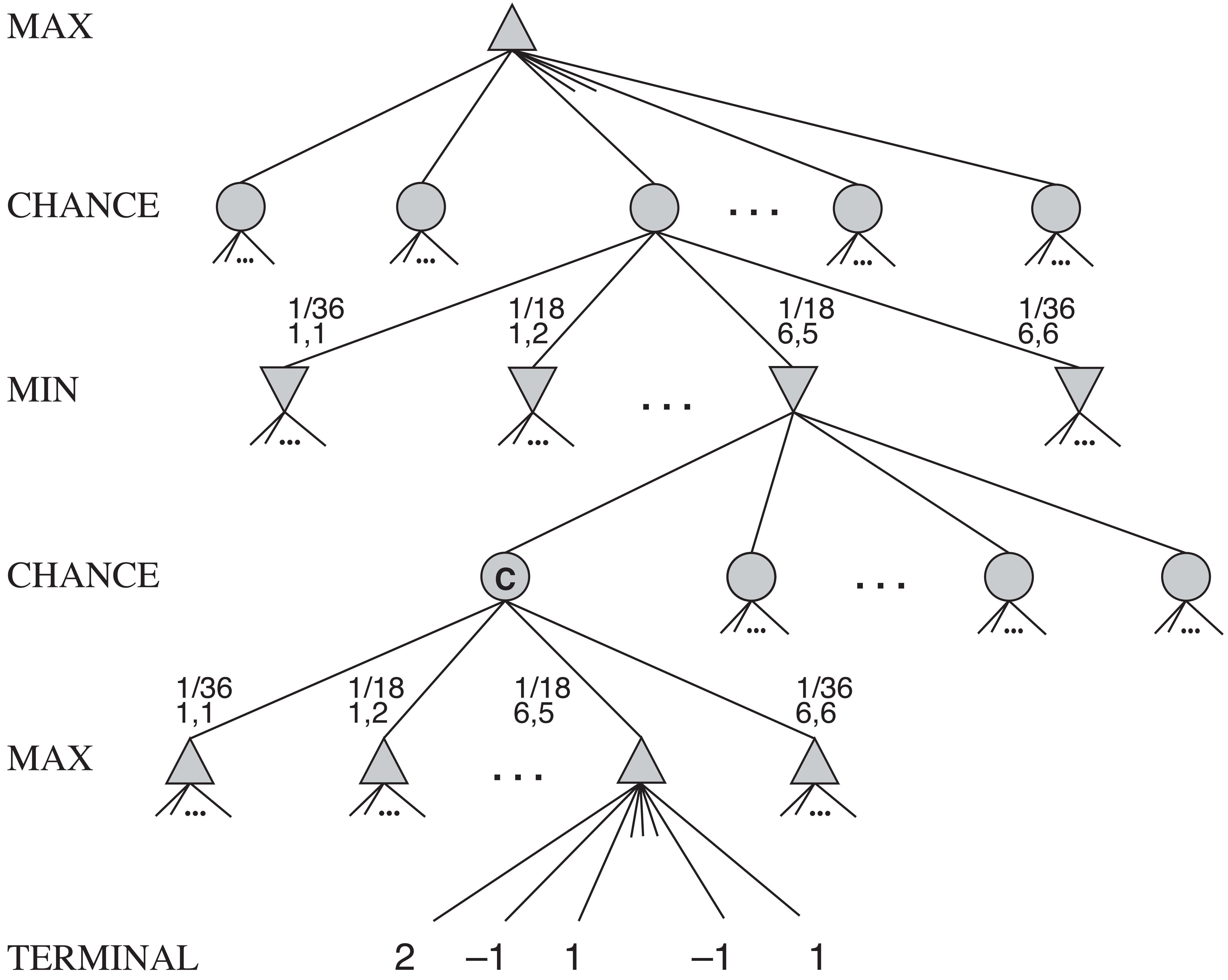
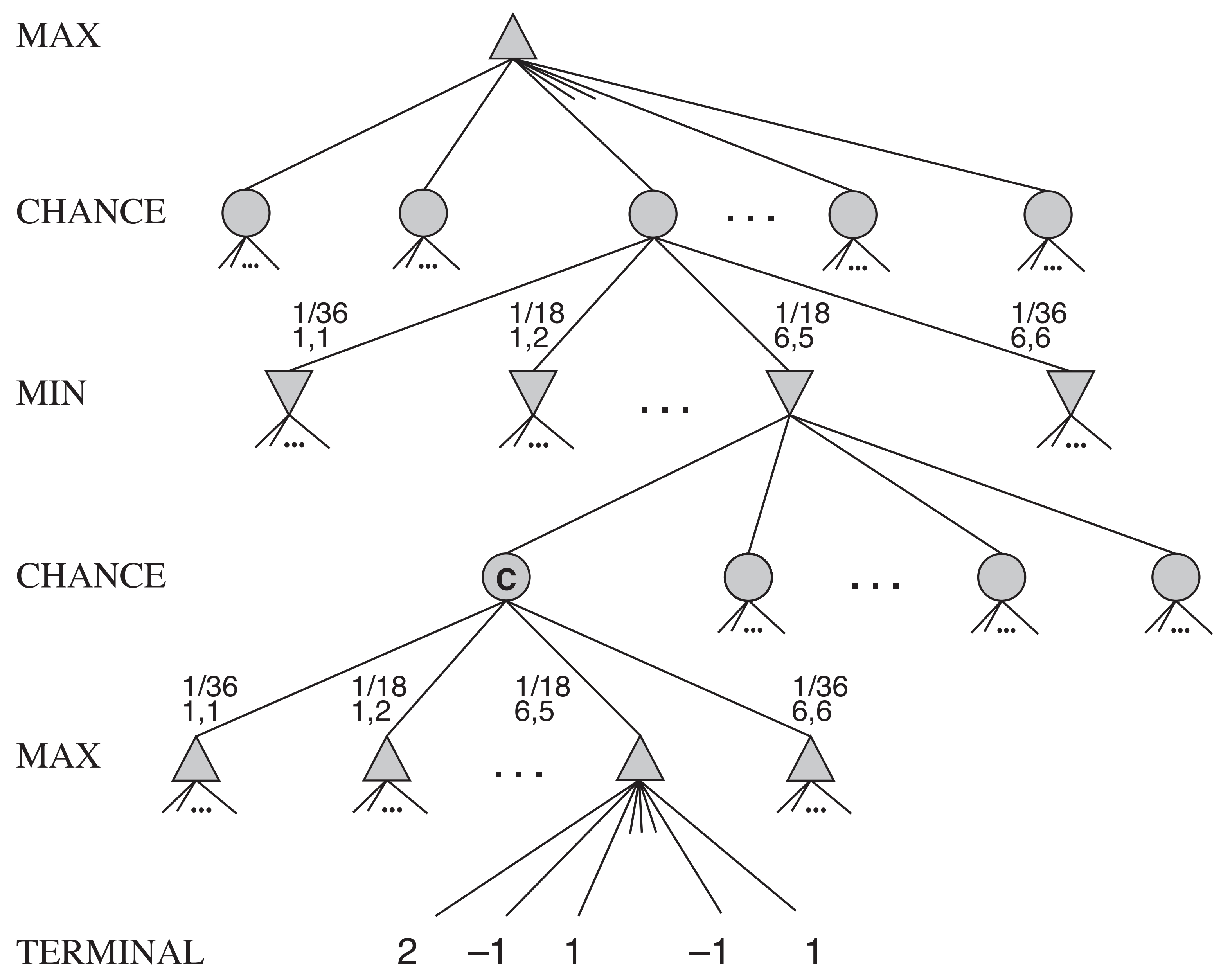
Expectiminimax

- New player type: Chance

$$\textsc{Expectinimimax}(s) =$$

$$\begin{cases} \textsc{Utility}(s) & \text{if } \textsc{Terminal-Test}(s) \\ \max_a \textsc{Expectiminimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{Max} \\ \min_a \textsc{Expectiminimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{Min} \\ \sum_r \Pr(r)\textsc{Expectiminimax}(\textsc{Result}(s,r)) & \text{if } \textsc{Player}(s) = \textsc{Chance} \end{cases}$$


MAX
CHANCE
MIN
CHANCE
MAX
TERMINAL
1/36
1,1
1/18
1,2
1/18
6,5
1/36
6,6
C
1/36
1,1
1/18
1,2
1/18
6,5
1/36
6,6
2
–1
1
–1
1



$$\text{EXPECTIMINIMAX}(s) =$$

$$\begin{cases} \text{UTILITY}(s) & \text{if TERMINAL-TEST}(s) \\ \max_a \text{EXPECTIMINIMAX}(\text{RESULT}(s,a)) & \text{if PLAYER}(s) = \text{MAX} \\ \min_a \text{EXPECTIMINIMAX}(\text{RESULT}(s,a)) & \text{if PLAYER}(s) = \text{MIN} \\ \sum_r \Pr(r)\text{EXPECTIMINIMAX}(\text{RESULT}(s,r)) & \text{if PLAYER}(s) = \text{CHANCE} \end{cases}$$

Pruning Chance Nodes

- Bounds on true utility function -> bounds on expectation
- -10 <= Utility <= 10
- uniform probability die:

?	?	?	?	?	?

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	10	?	?	?	?	?	
Best case	10	10	10	10	10	10	expectation 10
Worst case	10	-10	-10	-10	-10	-10	expectation -6.67

Pruning Chance Nodes

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	10	10	?	?	?	?	
Best case	10	10	10	10	10	10	expectation 10
Worst case	10	10	-10	-10	-10	-10	expectation -3.33

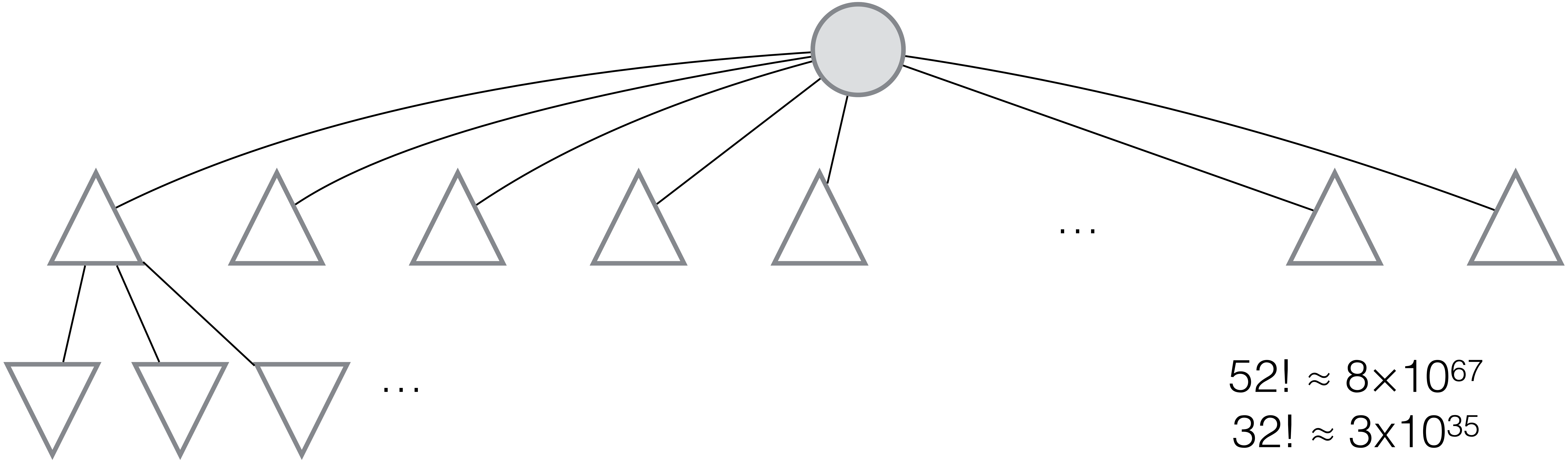
Pruning Chance Nodes

- Bounds on true utility function -> bounds on expectation
- -10 <= Utility <= 10
- uniform probability die:

	10	10	-10	?	?	?	
Best case	10	10	-10	10	10	10	expectation 6.67
Worst case	10	10	-10	-10	-10	-10	expectation -3.33

Partial Observations

- One approach: simulate perfect information with Chance nodes



$52! \approx 8 \times 10^{67}$

$32! \approx 3x10^{35}$

$22! \approx 4x10^{26}$

Summary

- Minimax logic works for any move ordering
- Expectiminimax adds Chance “player” and uses expected value
- Strategy for handling partial information