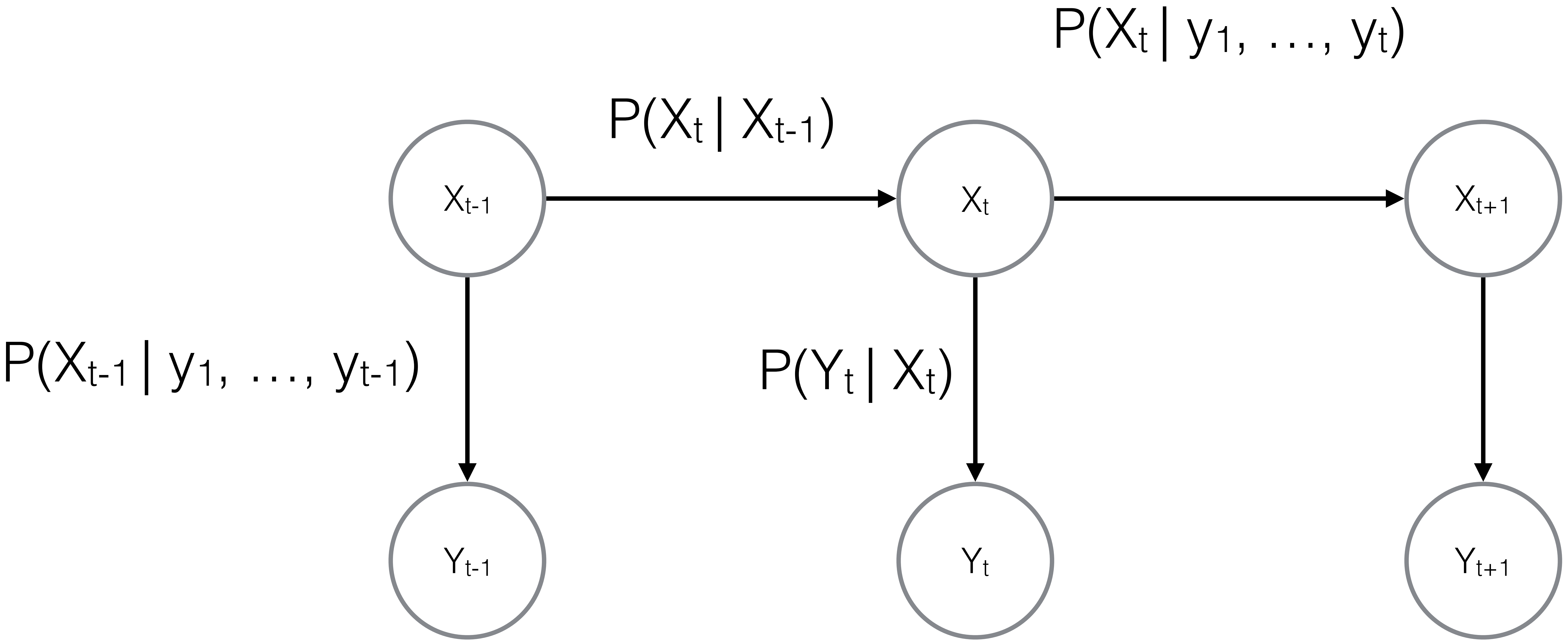
Particle Filters

Particle Filters

- Sample-based approximation of full inference
- Plan
  - Review full inference for tracking
  - Particle filter approximation

Inference for Live Tracking



Inference for Live Tracking

$P(X_t \,|\, y_1, \ldots, y_t)$

$P(Y_t \,|\, X_t)\; P(X_t \,|\, X_{t-1})\; P(X_{t-1} \,|\, y_1, \ldots, y_{t-1})$

$P(Y_t \,|\, X_t)\; P(X_t, X_{t-1} \,|\, y_1, \ldots, y_{t-1})$

$P(Y_t, X_t, X_{t-1} \,|\, y_1, \ldots, y_{t-1})$

$$P(Y_t = y, X_t \,|\, y_1, \ldots, y_{t-1}) = \sum_X P(Y_t = y, X_t, X_{t-1}{=}x \,|\, y_1, \ldots, y_{t-1})$$

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$$P(X_t \,|\, y_1, \ldots, y_t) = \frac{P(Y_t = y, X_t \,|\, y_1, \ldots, y_{t-1})}{\sum_X P(Y_t = y, X_t{=}x \,|\, y_1, \ldots, y_{t-1})}$$

Inference for Live Tracking

$$P(X_t \mid y_1, \ldots, y_t) = \frac{P(Y_t = y, X_t \mid y_1, \ldots, y_{t-1})}{\sum_X P(Y_t = y, X_t \mid y_1, \ldots, y_{t-1})}$$

transition probability

$$P(X_t \mid y_1, \ldots, y_t) \propto \sum_X P(y_t \mid X_t)\ P(X_t \mid X_{t-1}{=}x)\ P(X_{t-1}{=}x \mid y_1, \ldots, y_{t-1})$$

observation probability

computed from previous time step

Computational Cost

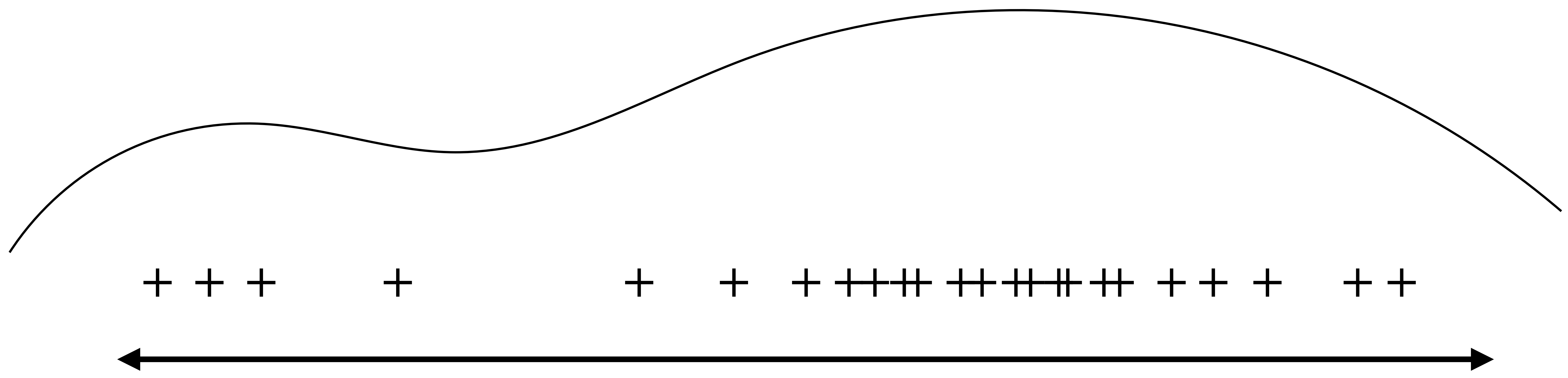
$$\sum_x P(Y_t \mid X_t) P(X_t \mid X_{t-1}=x) P(X_{t-1}=x \mid y_1, \ldots, y_{t-1})$$

For all X_t states, sum over all possible X_{t-1} states

cost: $|X_t| \, |X_{t-1}|$

$|X|$ is how many states in the environment. Could be huge!

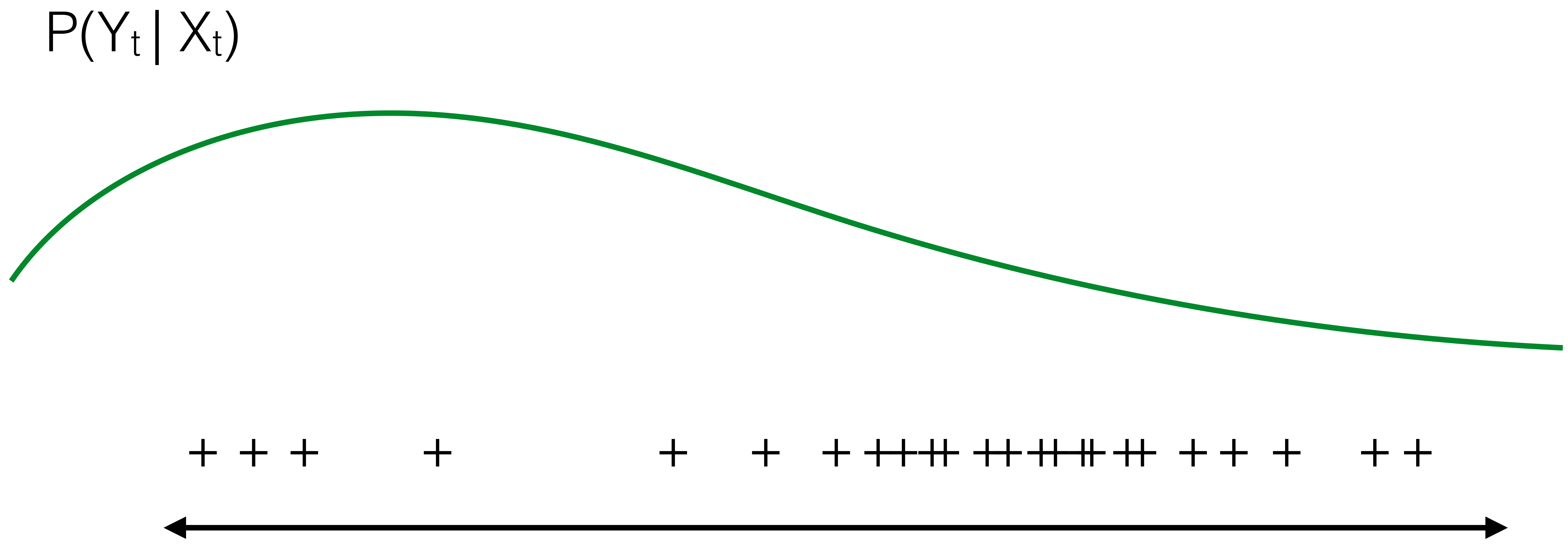
Particle Filtering



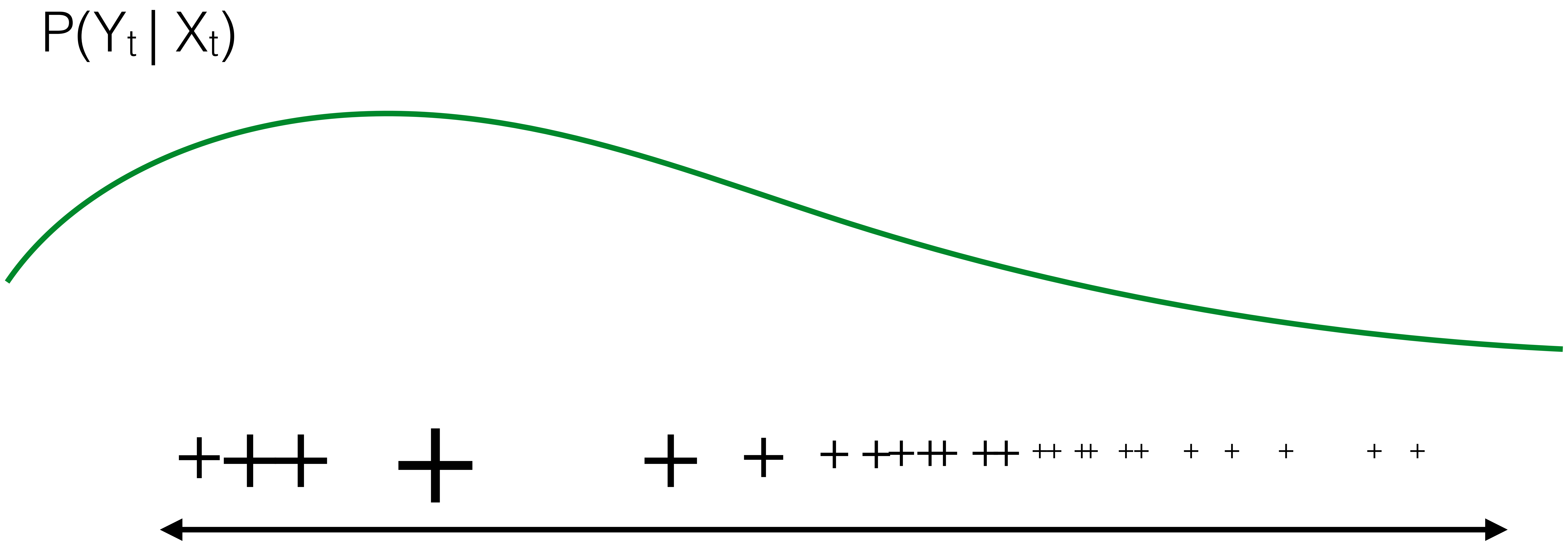
Step 1: Transition

$P(X_t \mid X_{t-1})$

Step 2: Reweighting

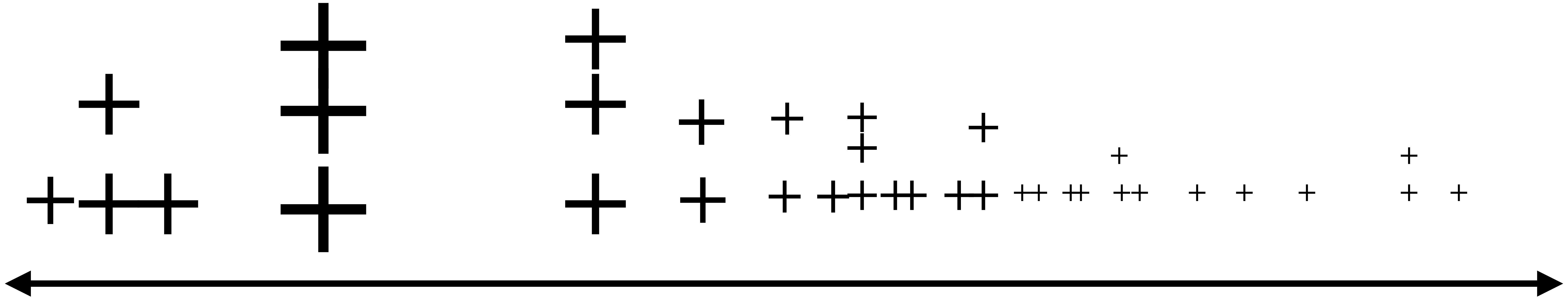


Step 2: Reweighting



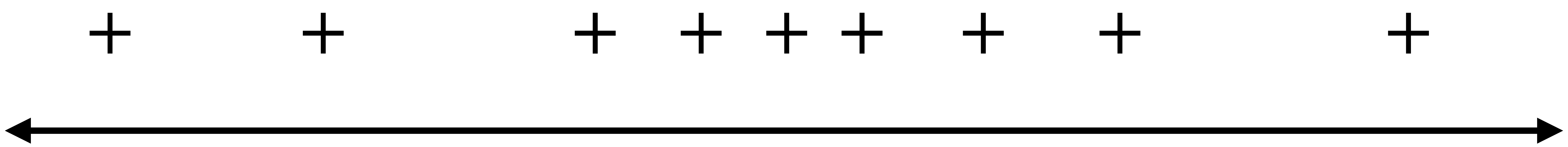
Step 3: Resampling

$P(Y_t \mid X_t)$



Step 3: Resampling

$P(Y_t \mid X_t)$



Particle Filtering

- Use **n** “particles” to represent distribution over hidden states
  - Transition: sample next state for each particle $O(k\ n)$
  - Evidence: weight samples based on evidence $O(n)$
  - Resample to generate a new distribution of particles $O(n \log n)$