First Order Logic

Introduction to AI
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Review

- Propositional logic syntax and semantics
- Inference in propositional logic: table, inference rules, resolution
- Horn clauses, forward/backward chaining

First Order Logic

- Terminology: predicates, arity, terms, functions, atoms
- Quantifiers: universal, existential
- Inference: differences from propositional logic

Terminology

- Predicates: functions that map variables to truth values
  - is_woman(Marge), is_mother_of(Lisa, Marge)
  - “is_woman” is a **predicate**, “Marge” is a **term/constant**, and “is_woman(Marge)” is a **formula**
  - Can have any **arity**: binary, 3-ary, 4-ary…n-ary
- Functions map from terms to terms: e.g., motherOf(Bart)→Marge

Simple Propositionalization

- Define propositional logic variables for every atomic formula
  - M1 = is_mother_of(Bart, Homer), M2 = is_mother_of(Lisa, Snowball), …

    W1 = is_woman(Marge), W2 = is_woman(Lisa), W3 = is_woman(Homer), …
- Perform inference as in propositional logic

Child	Mother	is_mother_of
Bart	Homer	F
Lisa	Snowball	F
Bart	Marge	T
Homer	Mona	T

Name	is_woman
Lisa	T
Marge	T
Homer	F
Bart	F

Quantifiers

- More general knowledge: e.g., “all mothers are people”
  - ∀ x, ∀y, is_mother_of(x, y) → is_person(y)
  - For all x and y, if y is x’s mother, then y is a person.
  - Universal quantifier: ∀
- Existential quantifier: ∃ (there exists)
  - ∀x, ∃y, is_mother_of(x, y)

Propositionalizing is Expensive

Lisa
Maggie
Marge
Bart
Homer

is_mother_of

Lisa
Maggie
Marge
Bart
Homer

is_mother_of(Lisa, Lisa)
is_mother_of(Lisa, Maggie)
is_mother_of(Lisa, Marge)
is_mother_of(Lisa, Bart)
is_mother_of(Lisa, Homer)
is_mother_of(Maggie, Lisa)
is_mother_of(Maggie, Maggie)
is_mother_of(Maggie, Marge)
is_mother_of(Maggie, Bart)
is_mother_of(Maggie, Homer)
is_mother_of(Marge, Lisa)
is_mother_of(Marge, Maggie)

Horn Clauses

$$pred_1(v_{11}, \ldots) \wedge pred_2(v_{21}, \ldots) \wedge \ldots \wedge pred_n(v_{n1}, \ldots) \rightarrow pred_r(v_{r1}, \ldots)$$

antecedent

consequent

Inference: Forward Chaining

- Input: set of known atoms
  Input: set of Horn clauses
  Input: target formula to prove (or disprove)

  - Implication where all **antecedents** (i.e., conditions) and **consequent** (i.e., result) are positive

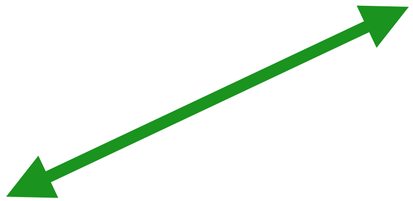
  - E.g., ∀x, ∀y, is_mother_of(x, y) → is_person(y)

  - Generate new knowledge using **generalized modus ponens**

Generalized Modus Ponens

E.g., ∀A, ∀B, ∀C, is_mother_of(A, B) ∧ is_mother_of(A, C) → sibling(B, C)

$pred_1(v_{11}, \ldots) \wedge pred_2(v_{21}, \ldots) \wedge \ldots \wedge pred_n(v_{n1}, \ldots) \rightarrow pred_r(v_{r1}, \ldots)$



KB

$pred_1(A, \ldots)$
$pred_2(B, \ldots)$
$pred_1(C, \ldots)$
…

Substitution

$v_{11} = A$
$v_{21} = C$
$v_{22} = A$
$v_{31} = B$
…

Generalized Modus Ponens

E.g., $\forall A, \forall B, \forall C, \text{is\_mother\_of}(A, B) \wedge \text{is\_mother\_of}(A, C) \rightarrow \text{sibling}(B, C)$

KB

is_mother_of(Marge, Maggie)
is_mother_of(Marge, Bart)

Substitution

A = Marge
B = Maggie
C = Bart

Handling Quantifiers

- Universal (∀): replace with any constant during substitution
- Existential (∃): replace existential variable with new constant

(Skolem) Constants

- Replace existentially quantified variables with arbitrary constant
  - ∃x, is_mother_of(x, Bart)
  - is_mother_of(C, Bart)

Forward Chaining, High Level

- For each sentence **r** in KB,
  - For each substitution **s** that satisfies the conditions of **r**
    - Add the consequent of **r** with substitution **s** to KB
    - Check if target sentence is in KB or if contradiction

Backward Chaining High-Level

- For target sentence
  - Look for substitution **s** that would imply target
    - Add unsatisfied clauses in antecedent of substituted sentence to frontier
    - Search for clauses that imply unsatisfied classes from frontier

Book Example

- “The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.” **Is West a criminal?**

- $\forall x, y, z, \text{American}(x) \land \text{Weapon}(y) \land \text{Sells}(x, y, z) \land \text{Hostile}(z) \Rightarrow \text{Criminal}(x)$,

- $\exists x, \text{Owns}(\text{Nono}, x) \land \text{Missile}(x)$
  - $\text{Owns}(\text{Nono}, \text{M1})$
  - $\text{Missile}(\text{M1})$

- $\forall x, \text{Missile}(x) \land \text{Owns}(\text{Nono}, x) \Rightarrow \text{Sells}(\text{West}, x, \text{Nono})$

- $\forall x, \text{Missile}(x) \Rightarrow \text{Weapon}(x)$

- $\forall x, \text{Enemy}(x, \text{America}) \Rightarrow \text{Hostile}(x)$

- $\text{American}(\text{West})$

- $\text{Enemy}(\text{Nono}, \text{America})$

Book Example: Forward Chaining

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- ∀x, Enemy(x, America) ⇒ Hostile(x)
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- **Weapon(y)**
- **Sells(West, y, z)**
- **Hostile(z)**

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- **$\text{Hostile}(z)$**

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First Order Logic

- Terminology: predicates, arity, terms, functions, atoms
- Quantifiers: universal, existential
- Inference: differences from propositional logic
  - Requires handling quantifiers, finding substitutions
  - Forward and Backward chaining