Logic

## Virginia Tech CS 4804

# Outline

- Propositional logic syntax and semantics
- Inference in propositional logic
- (Horn clauses, forward/backward chaining)

# Terminology

- **Knowledge base** (KB): a set of logical **sentences**
- Axioms are assumed true without proof
  - New sentences can be found through **inference**
  - Logic has syntax and semantics
- Logical sentences have truth
- A KB has possible worlds or models: instantiations of variables where all sentences are true, i.e., satisfy
- New sentences that all models also satisfy are entailed by KB
- Inference methods can be **sound**: only infer truly entailed sentences
  - and it can be **complete**: able to infer any sentence that is entailed

# Propositional Logic

- Atomic sentences are Boolean variables: X, Y, Z (**atoms**)
- Complex sentences combine simpler sentences via connectives
  - not, negation:  $\neg X$ ,  $\neg Y$
  - and, conjunction: X ^ Y
  - or, disjunction: X v Y
  - implication:  $X \Rightarrow Y$
  - double implication, biconditional: X ⇔ Y

**literal**: X,  $\neg$ X, Y,  $\neg$ Y, Z,  $\neg$ Z !X X && Y provable: X ⊢ Y XIIY • entails:  $X \models Z$ 

## Inference

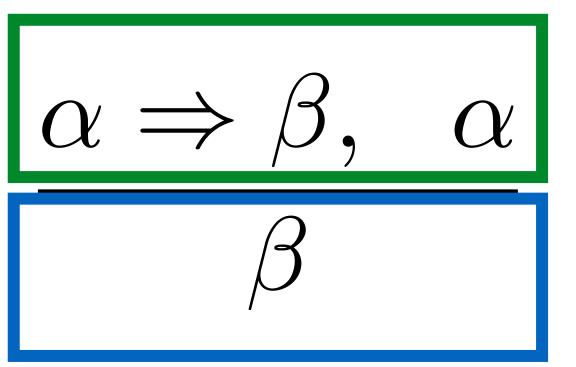
- Naive approach: evaluate full truth table
  - complete and sound inference but expensive
- Better approach: execute **inference rules** on KB: theorem proving

## • Given KB with prop. logic sentences, check if sentence α is true

KB: X ^ Y	X = F	X = T
Y = F	F	F
Y = T	F	Т

## Inference Rules

- Inference rules preserve **logical equivalence**
- Validity: a sentence is valid if it is true for all possible worlds
- Satisfiability: a sentence is satisfiable if it is true for at least one possible world
- Modus ponens: •

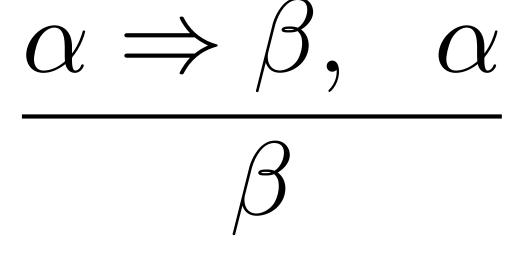


## in KB

new sentence

## 

- Validity: a sentence is valid if it is true for all possible worlds
- possible world
- Modus ponens: •



**And-elimination**: ullet



**Satisfiability**: a sentence is satisfiable if it is true for at least one



Category

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Category:Rules of inference

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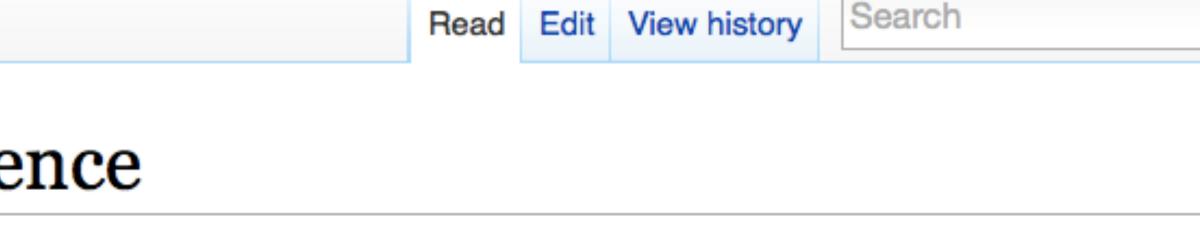
The main article for this category is **Rules of inference**.

The concepts described in articles in this category may be also expressed in terms of arguments, or theorems. Very often the same concept is in more than one of these categories, expressed a different way and sometimes with a different name.

## Pages in category "Rules of inference"

The following 40 pages are in this category, out of 40 total. This list may not reflect recent changes (learn more).

D cont. Rule of inference Disjunction elimination Disjunction introduction \* Disjunctive syllogism List of rules of inference Distributive property Double negative elimination Α Ε Absorption (logic) Admissible rule Existential generalization Associative property Existential instantiation



## M cont.

- Modus ponens
- Modus tollens

## Ν

- Negation as failure
- Negation introduction

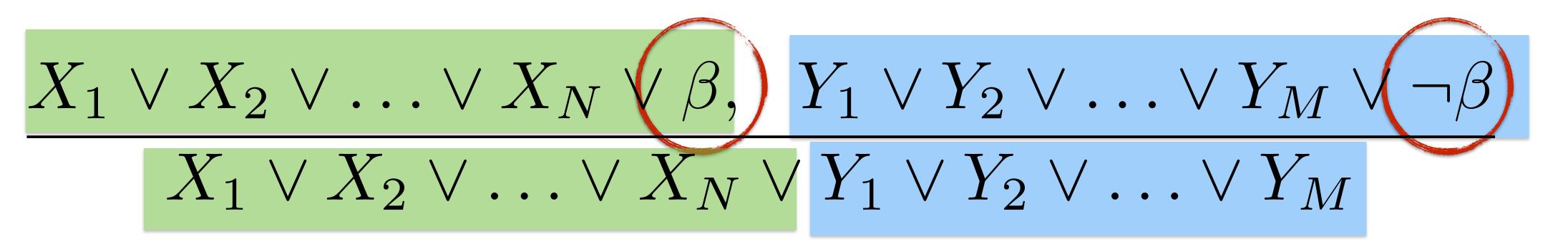
## R

Resolution (logic)



## Interence via Search

- States: knowledge bases Actions: apply inference rule, add new sentence to KB Goal: add target sentence a to KB
- Sound? Yes!
- Complete? Not necessarily

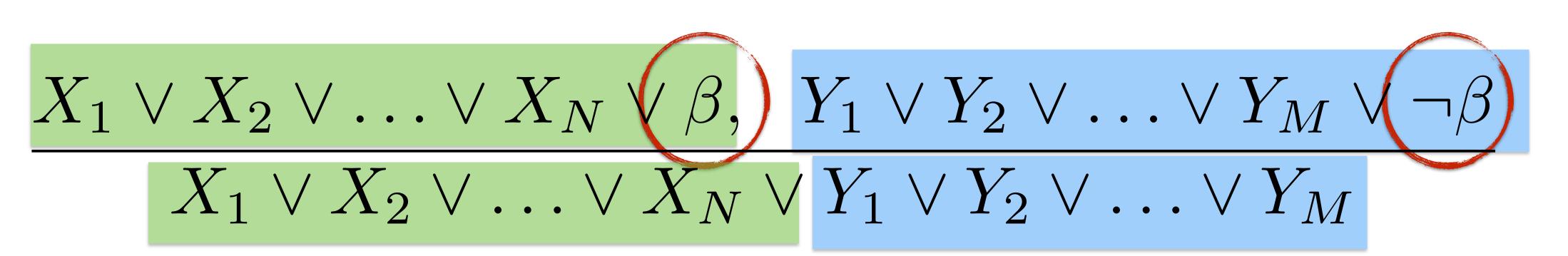


## Resolution

## Resolution

## Bob is a cat or lazy

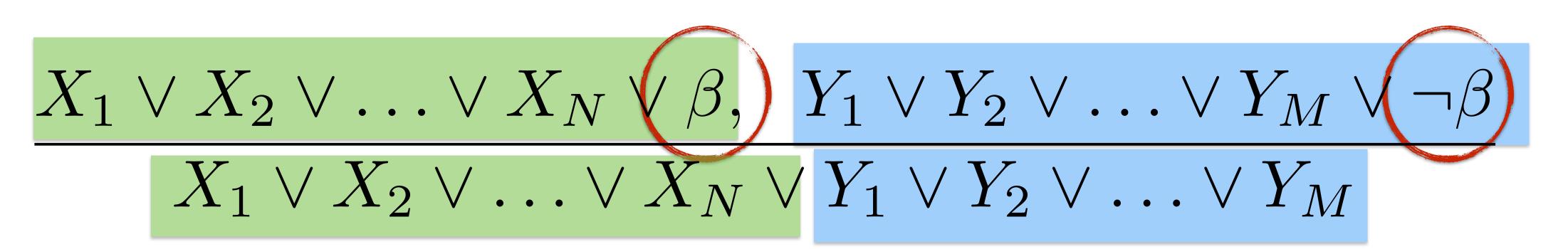
## Bob is a cat or a bird



## Bob is a bird or he is not lazy

## $A \Rightarrow B \equiv \neg A \lor B$

# R



# Resolution

## $\underline{A \Rightarrow B, A} \qquad \neg A \lor B, A$ R

# Conjunctive Normal Form

- Conjunction of **clauses** (disjunctions of literals)  $(X_1 \lor X_2 \lor X_3) \land (Y_1 \lor Y_2 \lor Y_3) \land (Z_1 \lor Y_2 \lor Y_3)$ 
  - $\alpha \Leftrightarrow \beta \equiv (\alpha \Rightarrow \beta) \land (\beta \Leftarrow \alpha)$  $\alpha \Rightarrow \beta \equiv \neg \alpha \lor \beta$  $\neg (\neg \alpha) \equiv \alpha$  $\neg (\alpha \land \beta) \equiv (\neg \alpha \lor \neg \beta)$  $\neg (\alpha \lor \beta) \equiv (\neg \alpha \land \neg \beta)$  $(\alpha \land (\beta \lor \gamma)) \equiv (\alpha \land \beta) \lor (\alpha \land \gamma)$  $(\alpha \lor (\beta \land \gamma)) \equiv (\alpha \lor \beta) \land (\alpha \lor \gamma)$

$$\vee Y_2 \vee Y_3) \wedge (Z_1 \vee Z_2 \vee Z_3)$$

- positive literal
  - equivalent to conjunction implying an atom

 $\neg X_1 \lor \neg X_2$  $X_1 \land X_2$ 

forward-chaining

## Horn clauses

• Restricted logic. Sentences must be disjunctions with at most one

$$_{2} \lor \neg X_{3} \lor Y$$
$$_{2} \land X_{3} \Rightarrow Y$$

backward-chaining

# Summary

- Propositional logic syntax and semantics
- Horn clauses, forward/backward chaining

Inference in propositional logic: table, inference rules, resolution