Logic

Virginia Tech CS 4804

Outline

- Propositional logic syntax and semantics
- Inference in propositional logic
- (Horn clauses, forward/backward chaining)

Terminology

- **Knowledge base** (KB): a set of logical **sentences**
- **Axioms** are assumed true without **proof**
  - New sentences can be found through **inference**
  - Logic has **syntax** and **semantics**
- **Logical** sentences have **truth**
- A KB has **possible worlds** or **models**: instantiations of variables where all sentences are true, i.e., **satisfy**
- New sentences that all models also satisfy are **entailed** by KB
- Inference methods can be **sound**: only infer truly entailed sentences
  - and it can be **complete**: able to infer any sentence that is entailed

Propositional Logic

- Atomic sentences are Boolean variables: X, Y, Z (**atoms**)
- Complex sentences combine simpler sentences via **connectives**
  - **not, negation**: ¬X, ¬Y
  - **and, conjunction**: X ∧ Y
  - **or, disjunction**: X ∨ Y
  - **implication**: X ⇒ Y
  - **double implication, biconditional**: X ⇔ Y

!X

X && Y

X || Y

literal: X, ¬X, Y, ¬Y, Z, ¬Z

- **provable**: X ⊢ Y
- **entails**: X ⊨ Z

Inference

- Given KB with prop. logic sentences, check if sentence α is true
- Naive approach: evaluate full truth table
  - complete and sound inference but *expensive*
- Better approach: execute **inference rules** on KB: **theorem proving**

KB: X ^ Y	X = F	X = T
Y = F	F	F
Y = T	F	T

Inference Rules

- Inference rules preserve **logical equivalence**
- **Validity**: a sentence is valid if it is true for all possible worlds
- **Satisfiability**: a sentence is satisfiable if it is true for *at least one* possible world
- **Modus ponens**:

$$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$

in KB

new sentence

- inference rules preserve **logical equivalence**

- **Validity**: a sentence is valid if it is true for all possible worlds

- **Satisfiability**: a sentence is satisfiable if it is true for *at least one* possible world

- **Modus ponens**: $$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$

- **And-elimination**: $$\frac{\boxed{\alpha \wedge \beta}}{\boxed{\alpha}}$$ **in KB** / **new sentence**

Category:Rules of inference

From Wikipedia, the free encyclopedia

*The main article for this category is **Rules of inference**.*

The concepts described in articles in this category may be also expressed in terms of arguments, or theorems. Very often the same concept is in more than one of these categories, expressed a different way and sometimes with a different name.

Pages in category "Rules of inference"

The following 40 pages are in this category, out of 40 total. This list may not reflect recent changes (learn more).

- Rule of inference

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- List of rules of inference

A

- Absorption (logic)
- Admissible rule
- Associative property

D cont.

- Disjunction elimination
- Disjunction introduction
- Disjunctive syllogism
- Distributive property
- Double negative elimination

E

- Existential generalization
- Existential instantiation

M cont.

- Modus ponens
- Modus tollens

N

- Negation as failure
- Negation introduction

R

- Resolution (logic)

Inference via Search

- States: knowledge bases
  Actions: apply inference rule, add new sentence to KB
  Goal: add target sentence α to KB

- Sound? Yes!

- Complete? Not necessarily

Resolution

$$\frac{X_1 \vee X_2 \vee \ldots \vee X_N \vee \beta, \quad Y_1 \vee Y_2 \vee \ldots \vee Y_M \vee \neg\beta}{X_1 \vee X_2 \vee \ldots \vee X_N \vee Y_1 \vee Y_2 \vee \ldots \vee Y_M}$$

Resolution

Bob is a cat or lazy

Bob is a bird or he is not lazy

Bob is a cat or a bird

$$\frac{X_1 \vee X_2 \vee \ldots \vee X_N \vee \beta, \quad Y_1 \vee Y_2 \vee \ldots \vee Y_M \vee \neg\beta}{X_1 \vee X_2 \vee \ldots \vee X_N \vee Y_1 \vee Y_2 \vee \ldots \vee Y_M}$$

Resolution

$$A \Rightarrow B \equiv \neg A \vee B$$

$$\frac{A \Rightarrow B, \quad A}{B} \qquad \frac{\neg A \vee B, \quad A}{B}$$

$$\frac{X_1 \vee X_2 \vee \ldots \vee X_N \vee \beta, \quad Y_1 \vee Y_2 \vee \ldots \vee Y_M \vee \neg\beta}{X_1 \vee X_2 \vee \ldots \vee X_N \vee Y_1 \vee Y_2 \vee \ldots \vee Y_M}$$

Conjunctive Normal Form

- Conjunction of **clauses** (disjunctions of literals)

$$(X_1 \vee X_2 \vee X_3) \wedge (Y_1 \vee Y_2 \vee Y_3) \wedge (Z_1 \vee Z_2 \vee Z_3)$$

$$\alpha \Leftrightarrow \beta \equiv (\alpha \Rightarrow \beta) \wedge (\beta \Leftarrow \alpha)$$
$$\alpha \Rightarrow \beta \equiv \neg\alpha \vee \beta$$
$$\neg(\neg\alpha) \equiv \alpha$$
$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$$
$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$$
$$(\alpha \wedge (\beta \vee \gamma)) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$$
$$(\alpha \vee (\beta \wedge \gamma)) \equiv (\alpha \vee \beta) \wedge (\alpha \vee \gamma)$$

Horn clauses

- Restricted logic. Sentences must be disjunctions with at most one positive literal
  - equivalent to conjunction implying an atom

$$\neg X_1 \vee \neg X_2 \vee \neg X_3 \vee Y$$

$$X_1 \wedge X_2 \wedge X_3 \Rightarrow Y$$

forward-chaining

backward-chaining

Summary

- Propositional logic syntax and semantics
- Inference in propositional logic: table, inference rules, resolution
- Horn clauses, forward/backward chaining