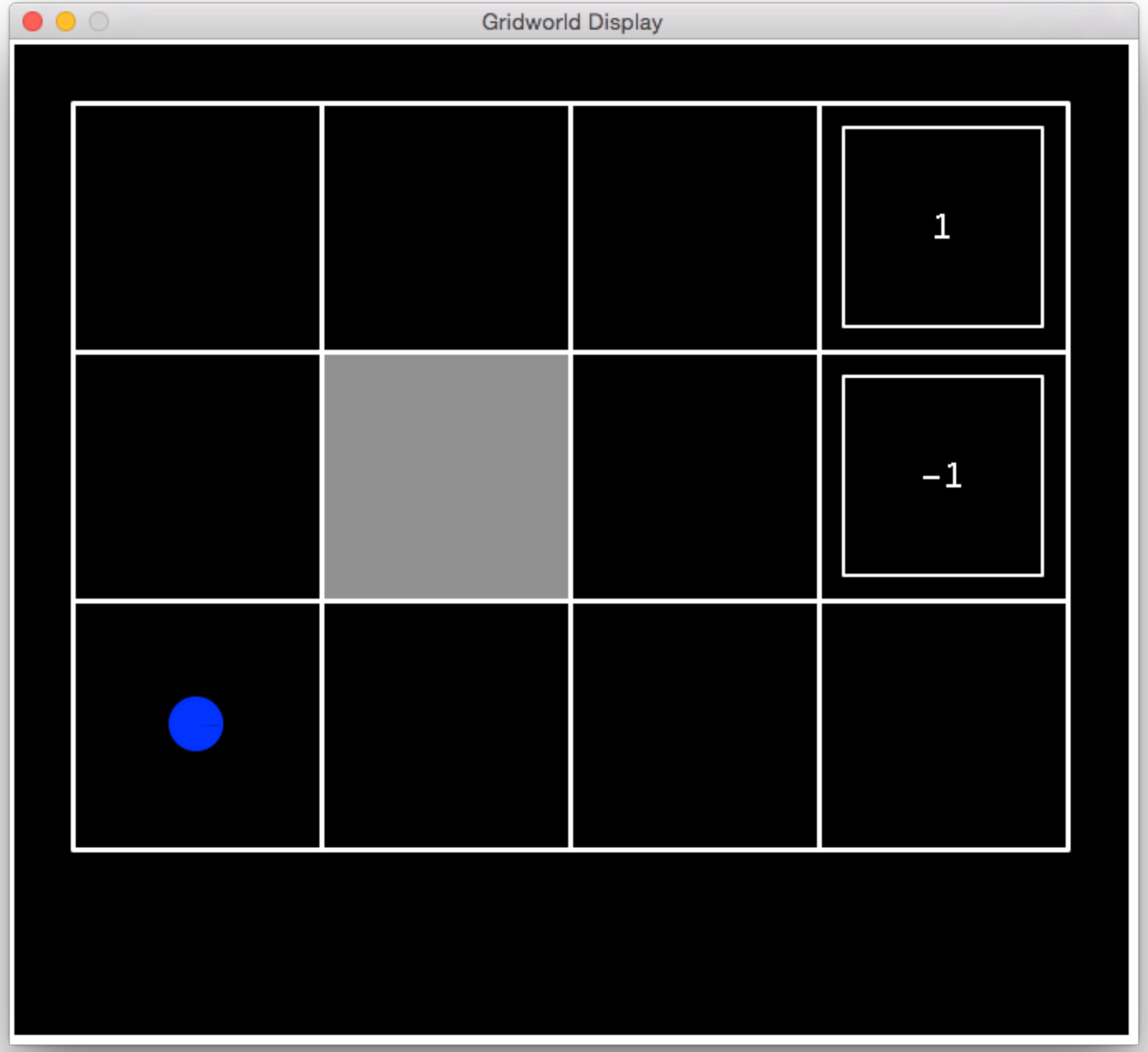
Markov Decision Processes

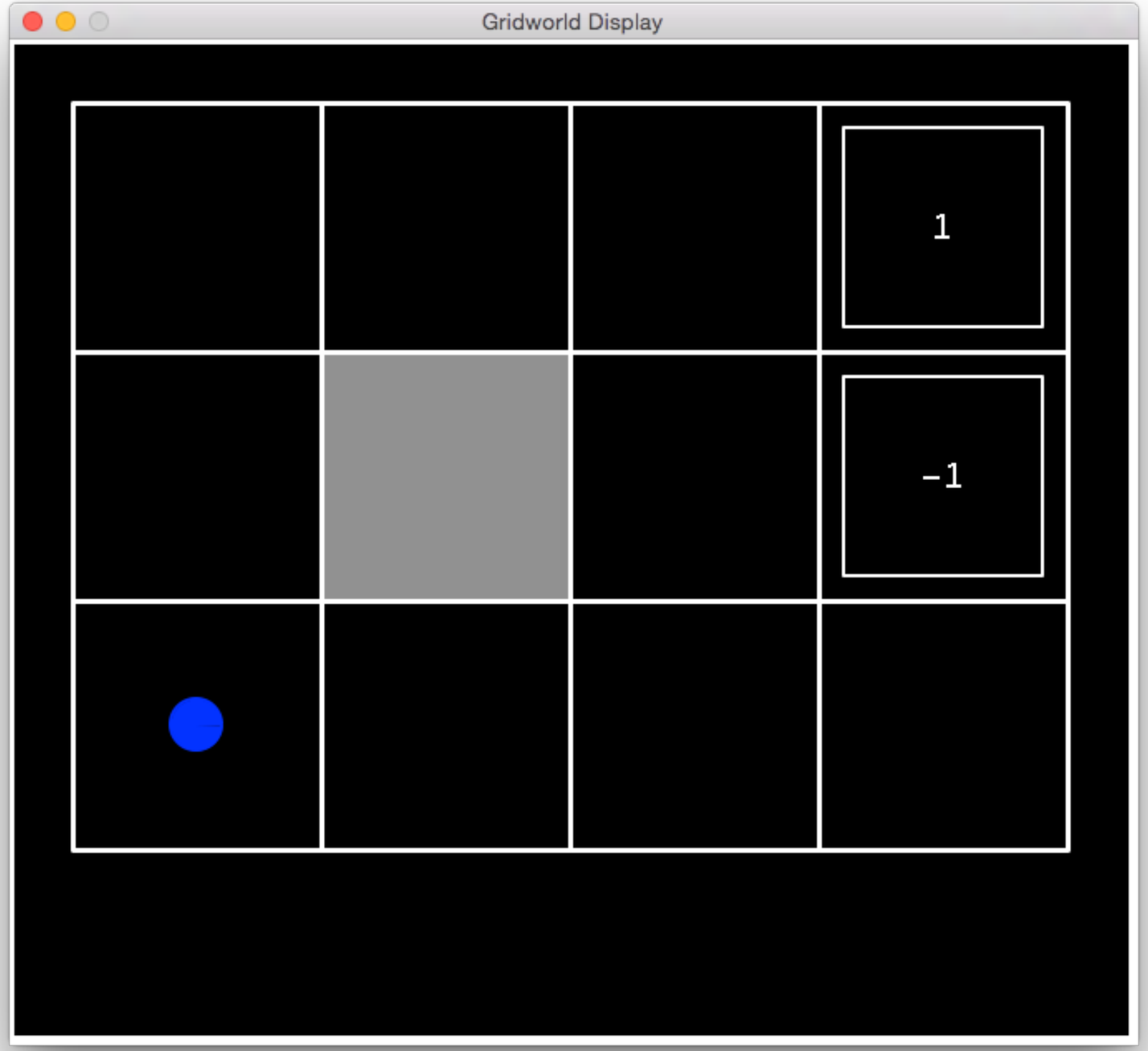
Virginia Tech CS5804
Spring 2015

Outline

- Markov decision process: richer environment representation
- Reward functions
- Optimizing policies via value iteration


Gridworld Display
1
-1

collect reward



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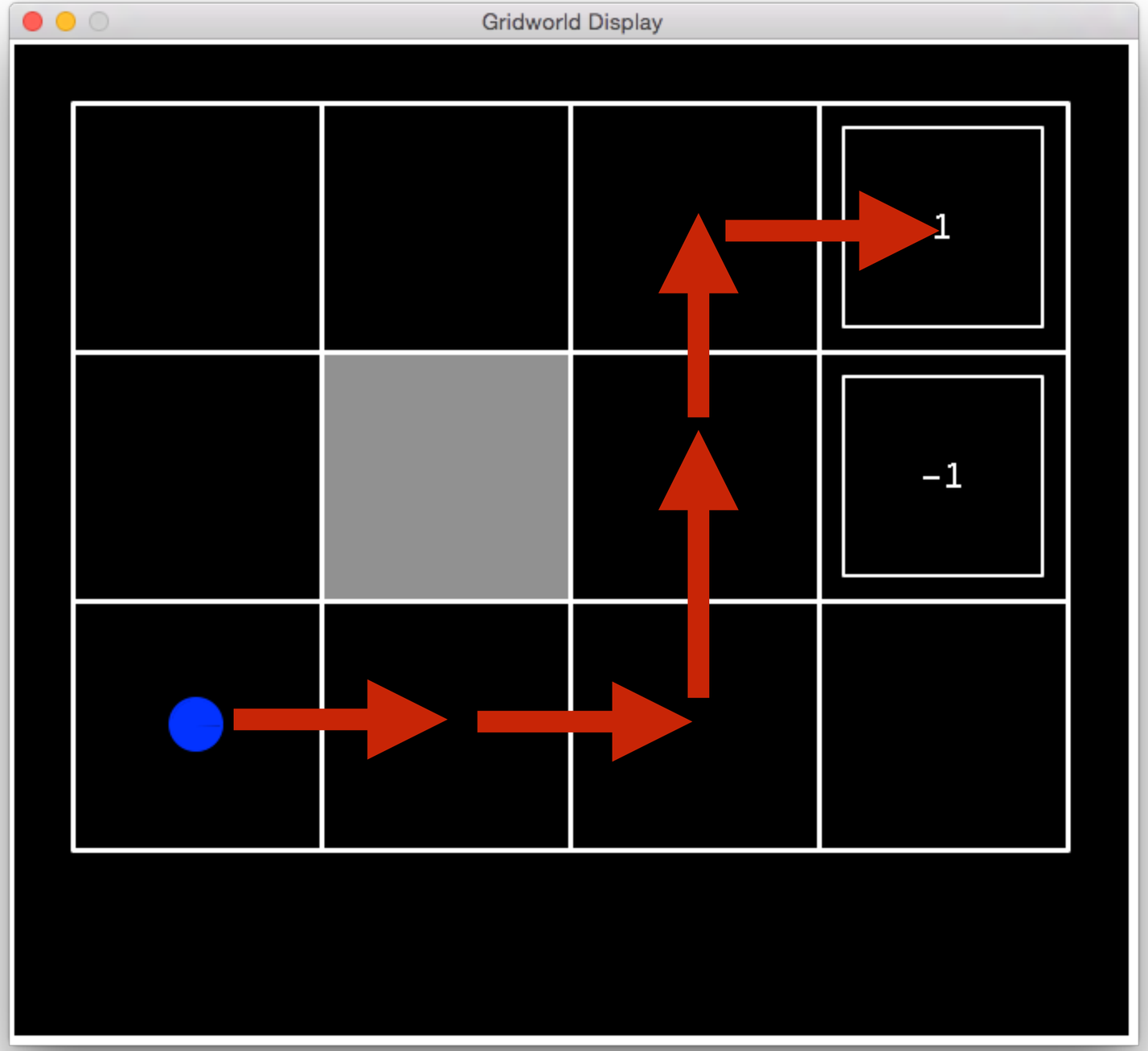

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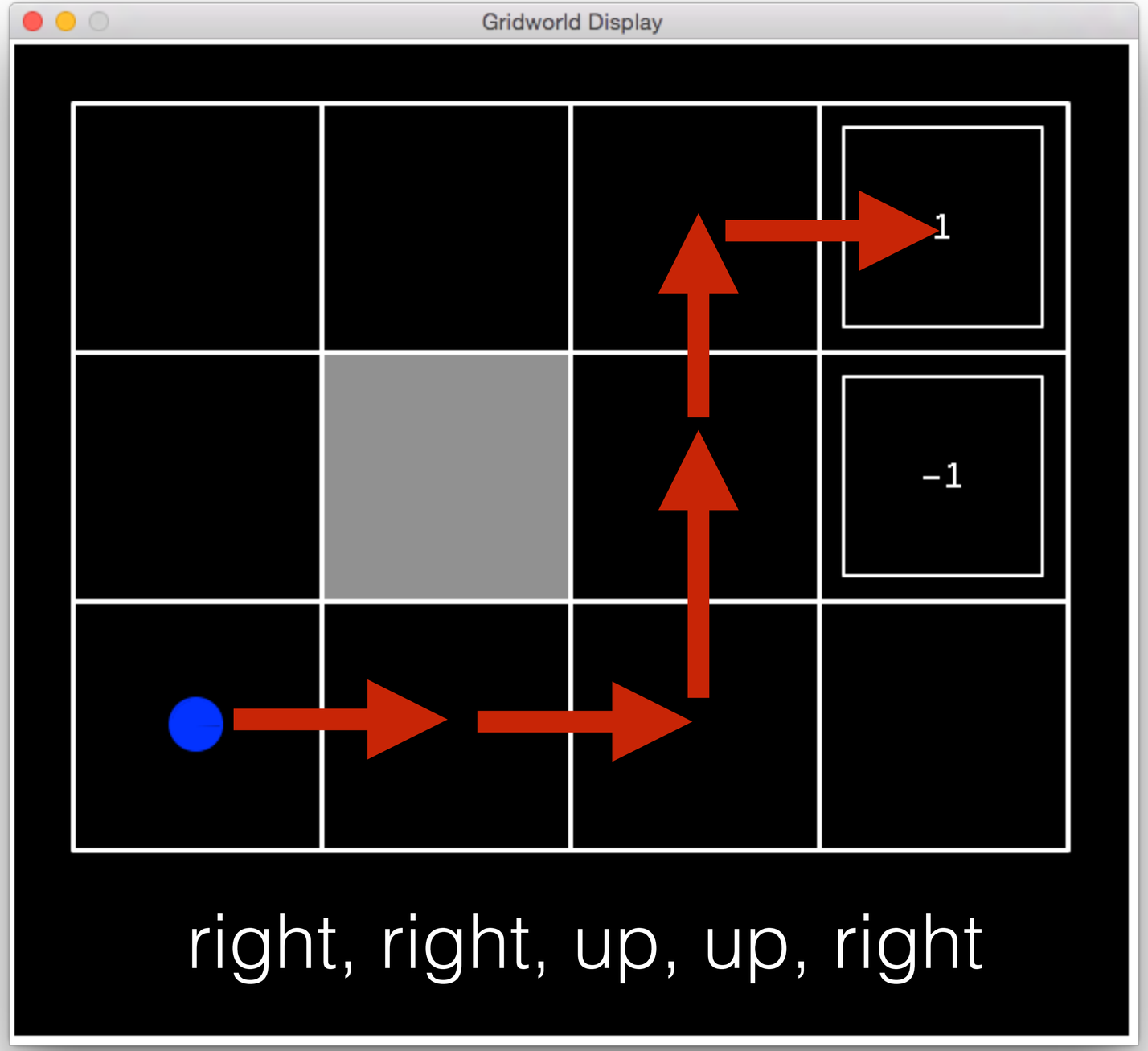

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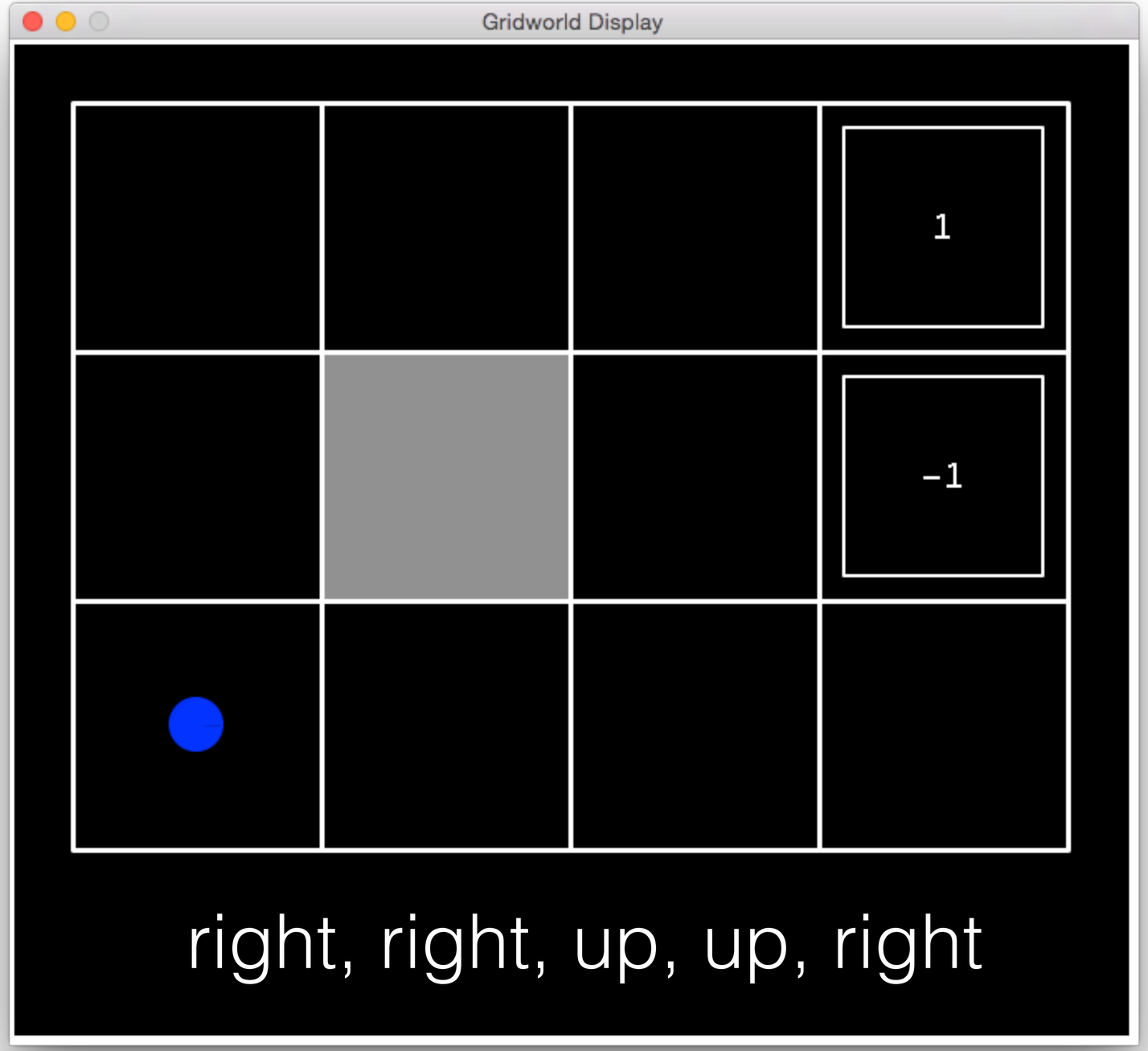
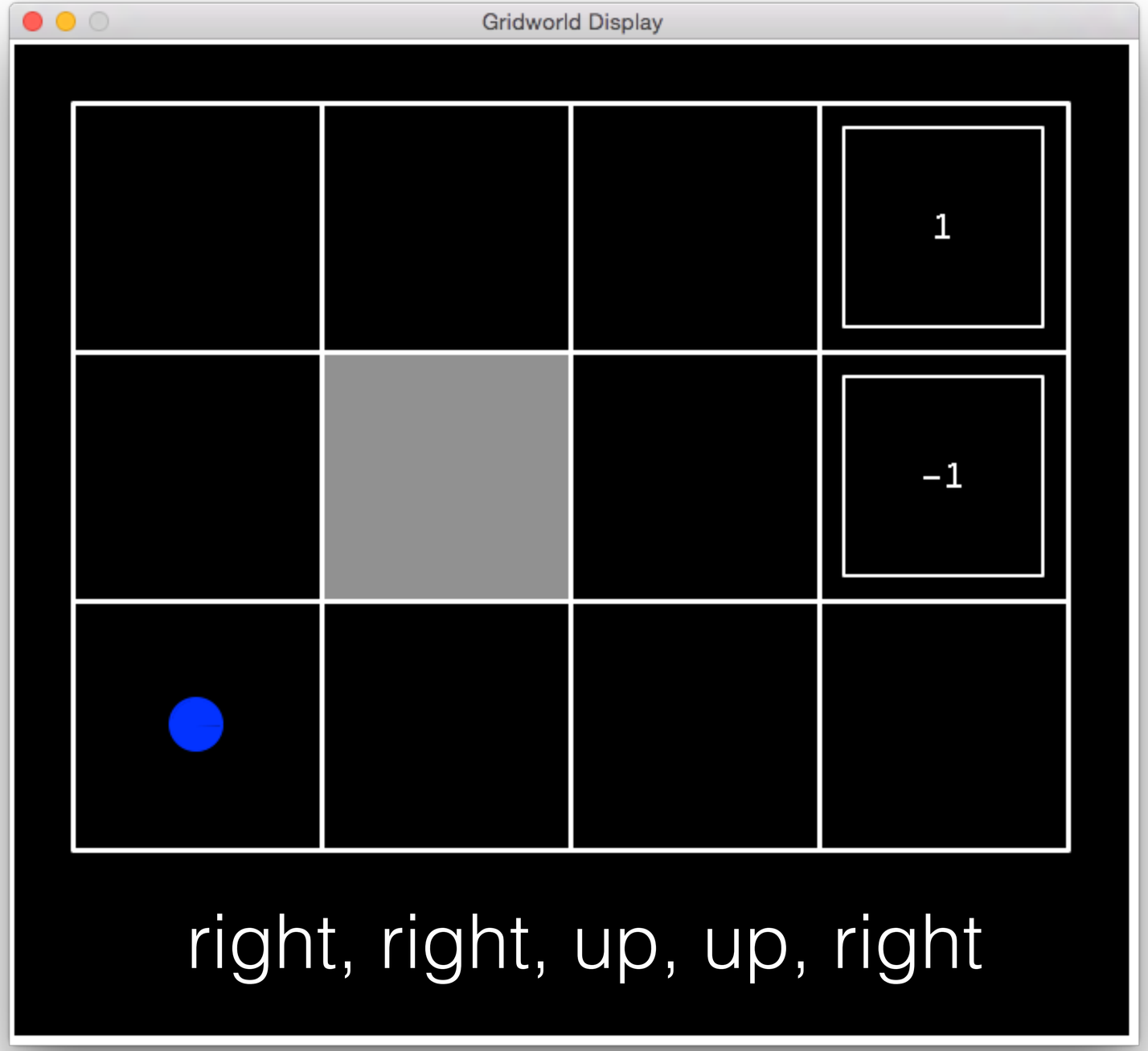


collect reward

stochastic transitions



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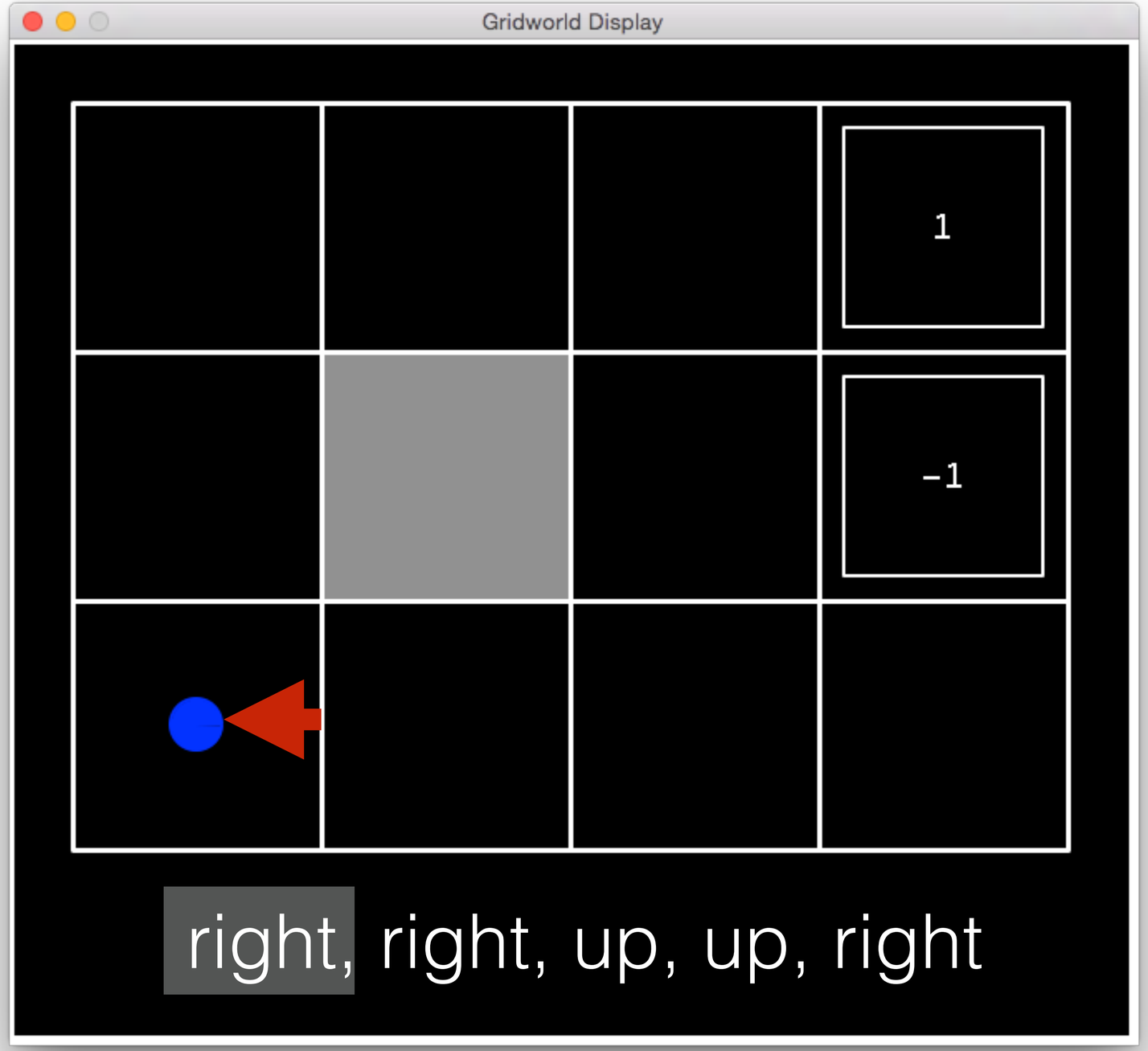

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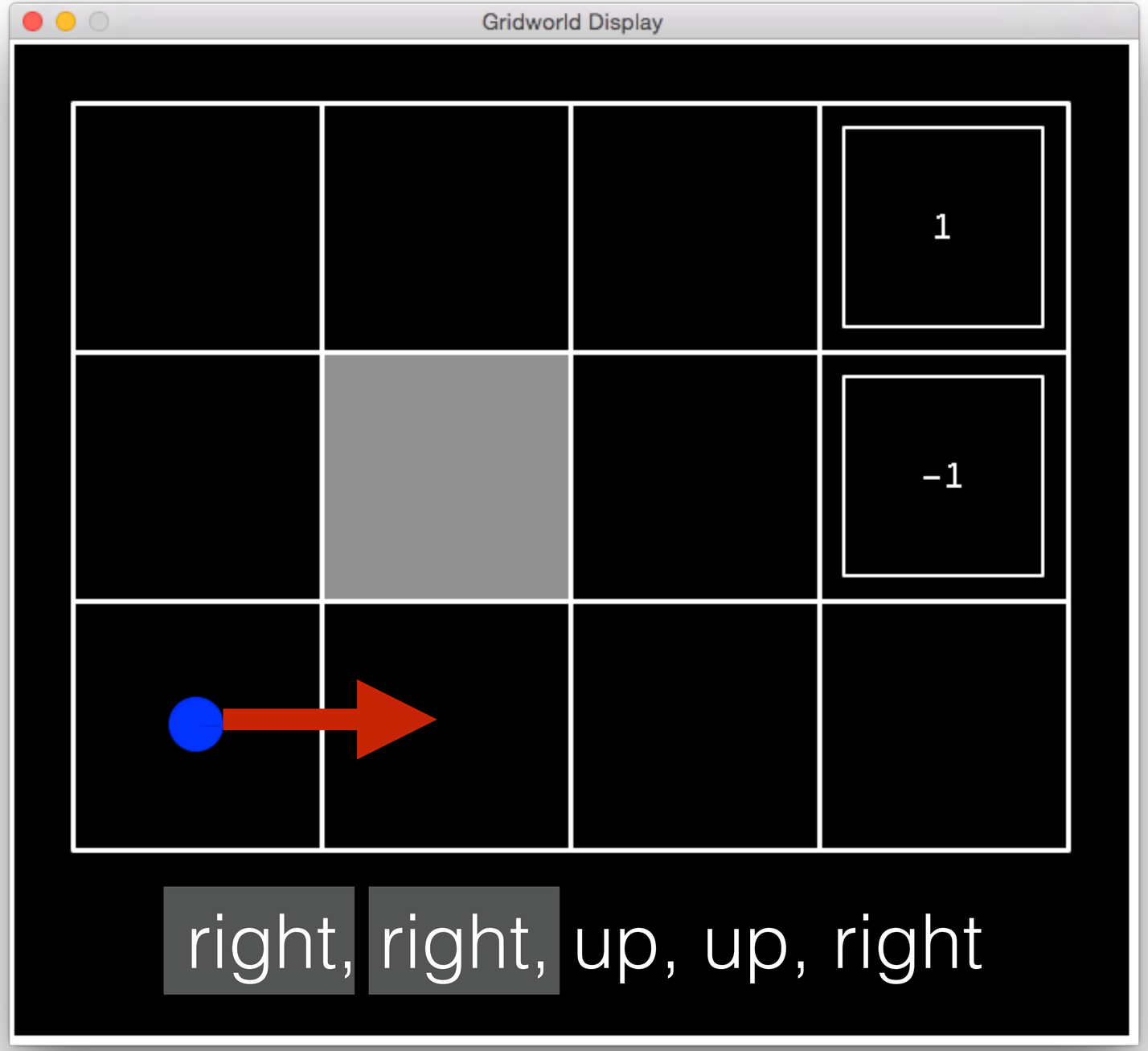

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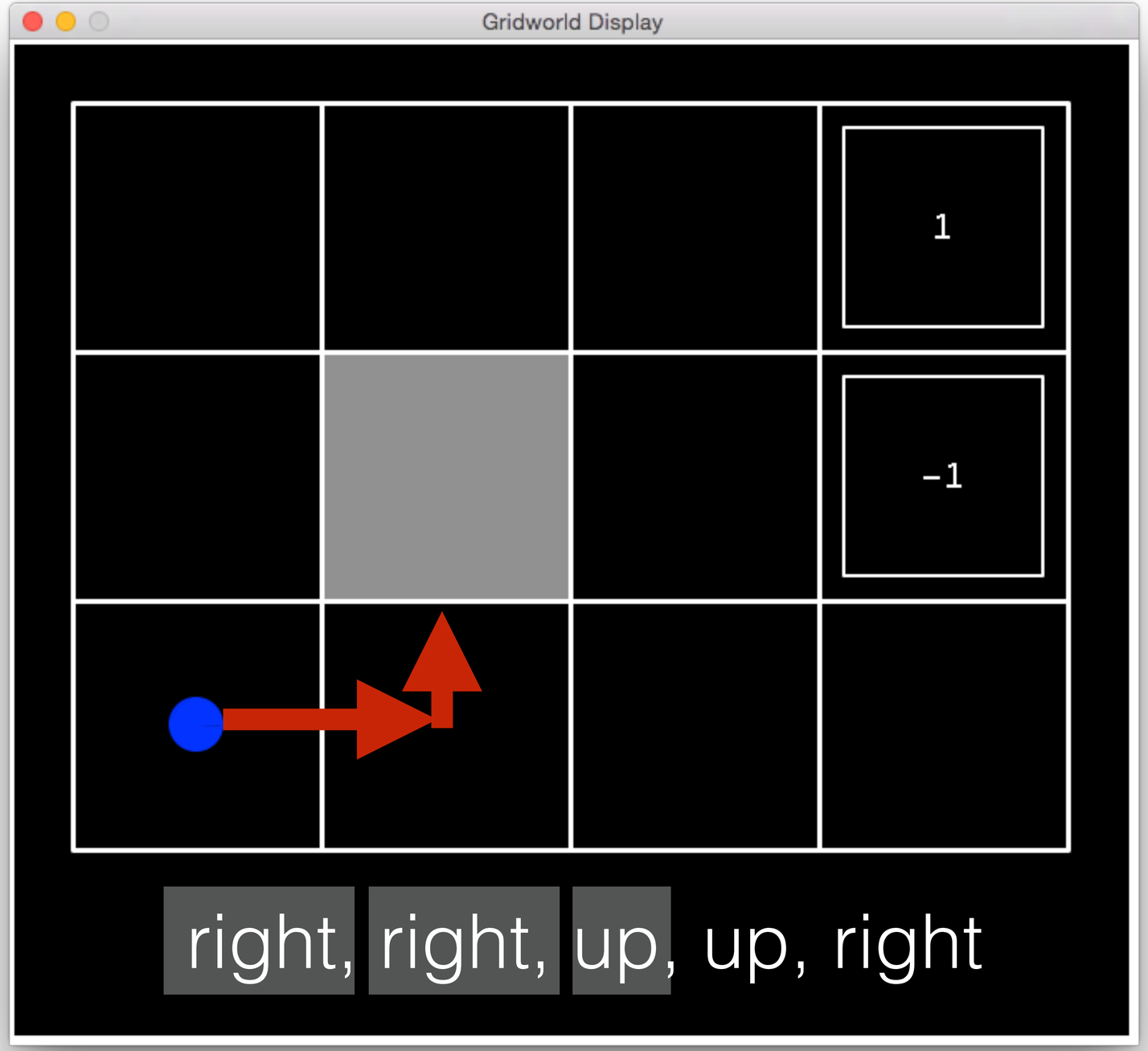

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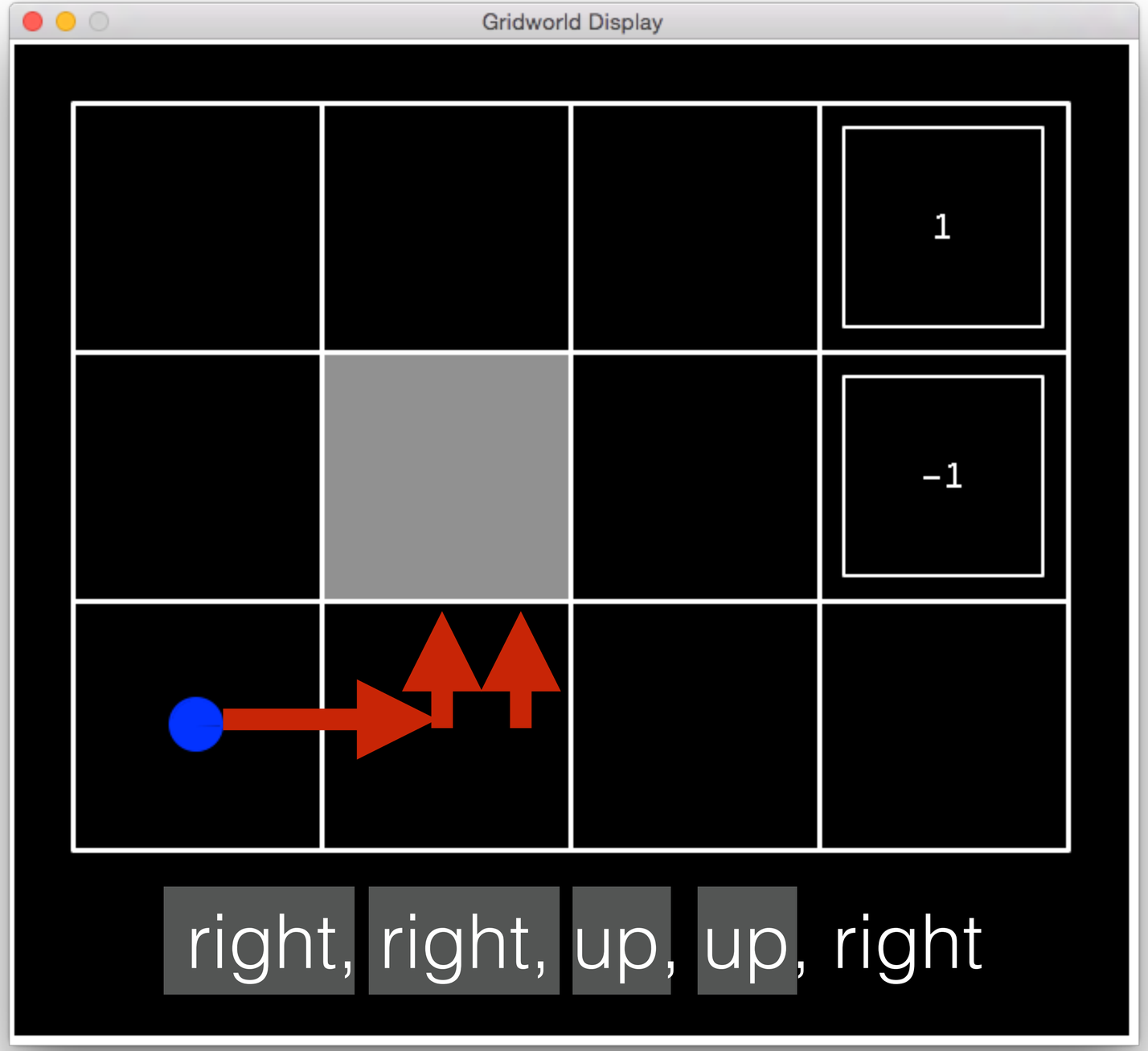

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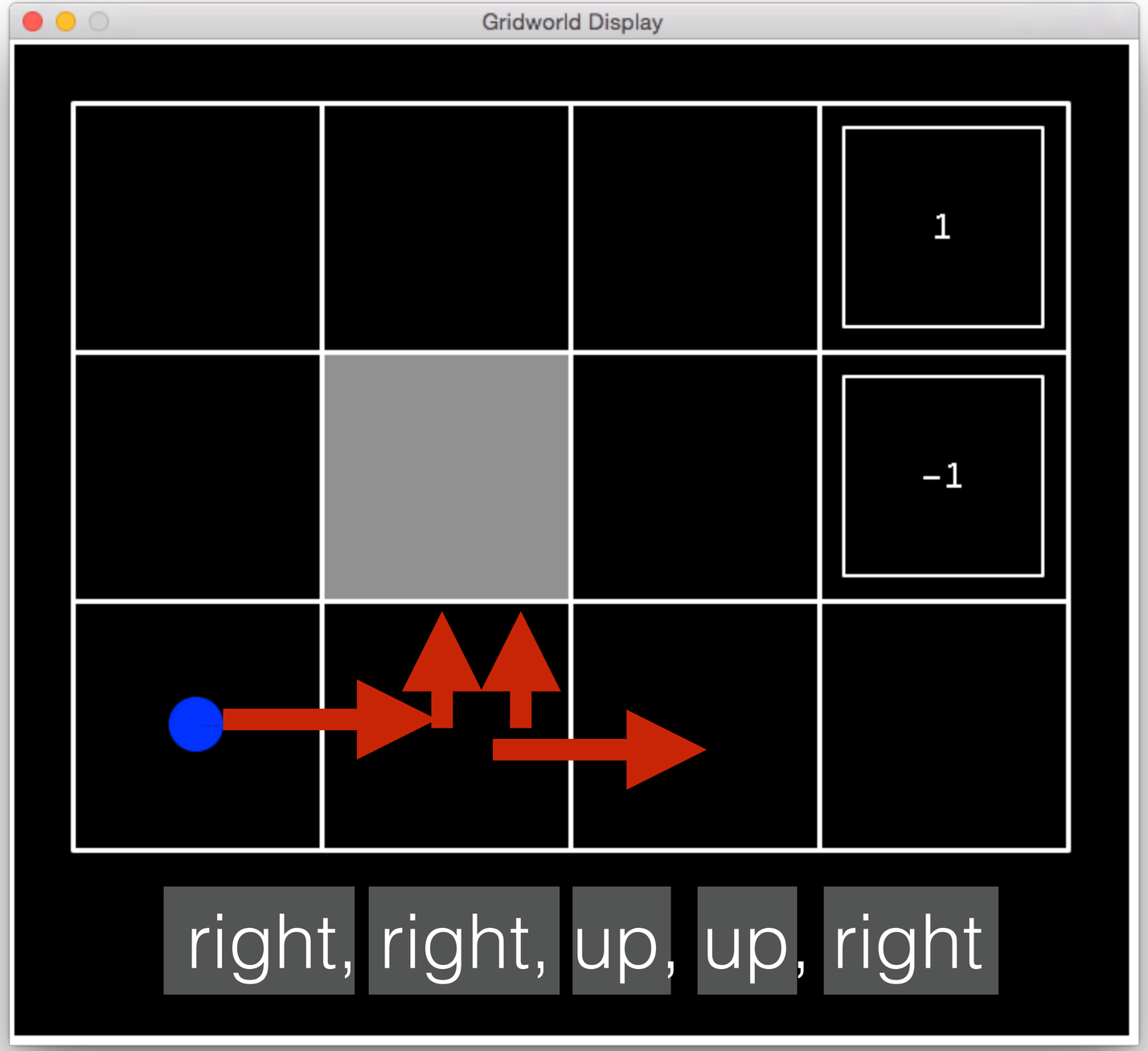

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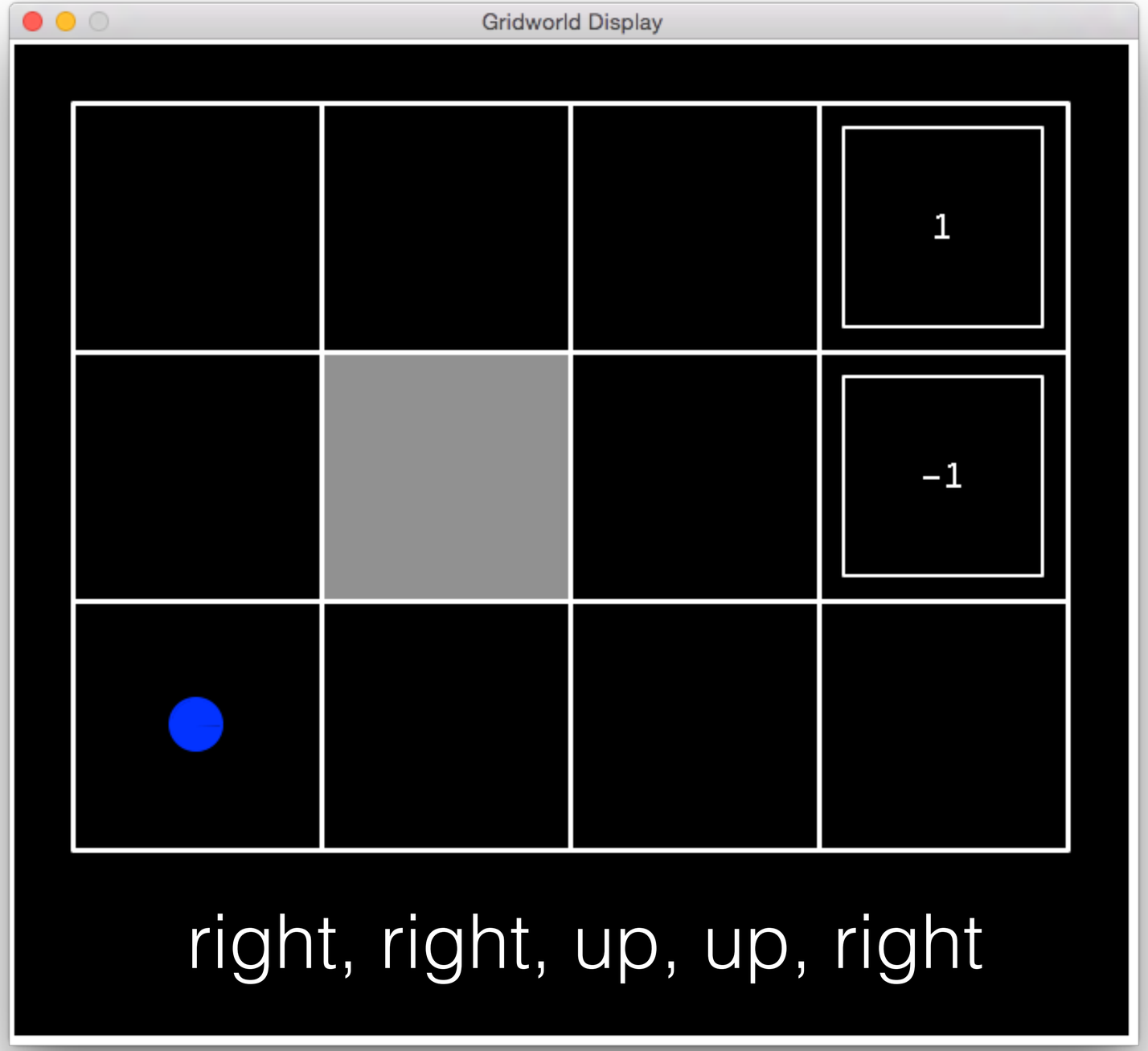

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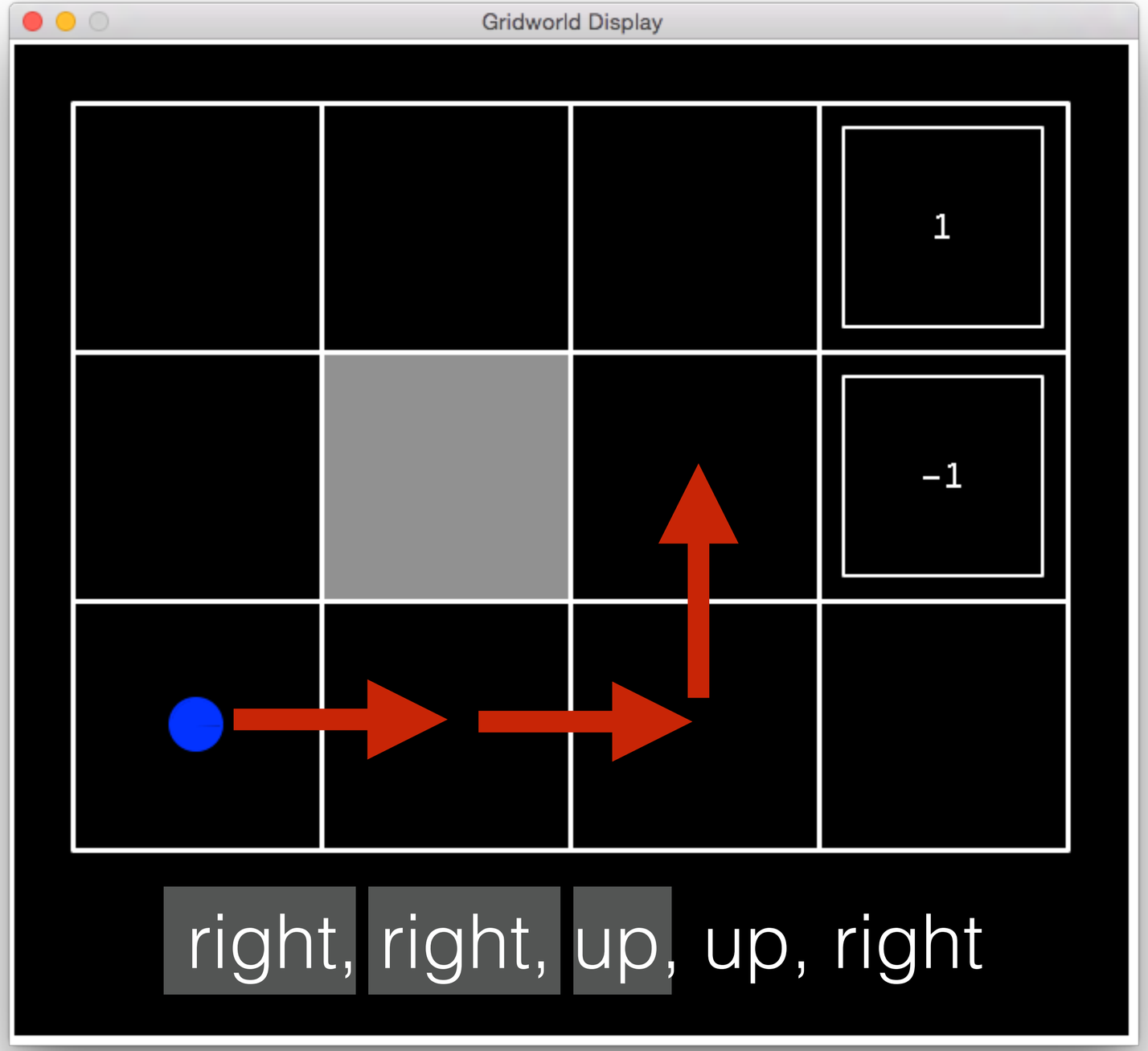

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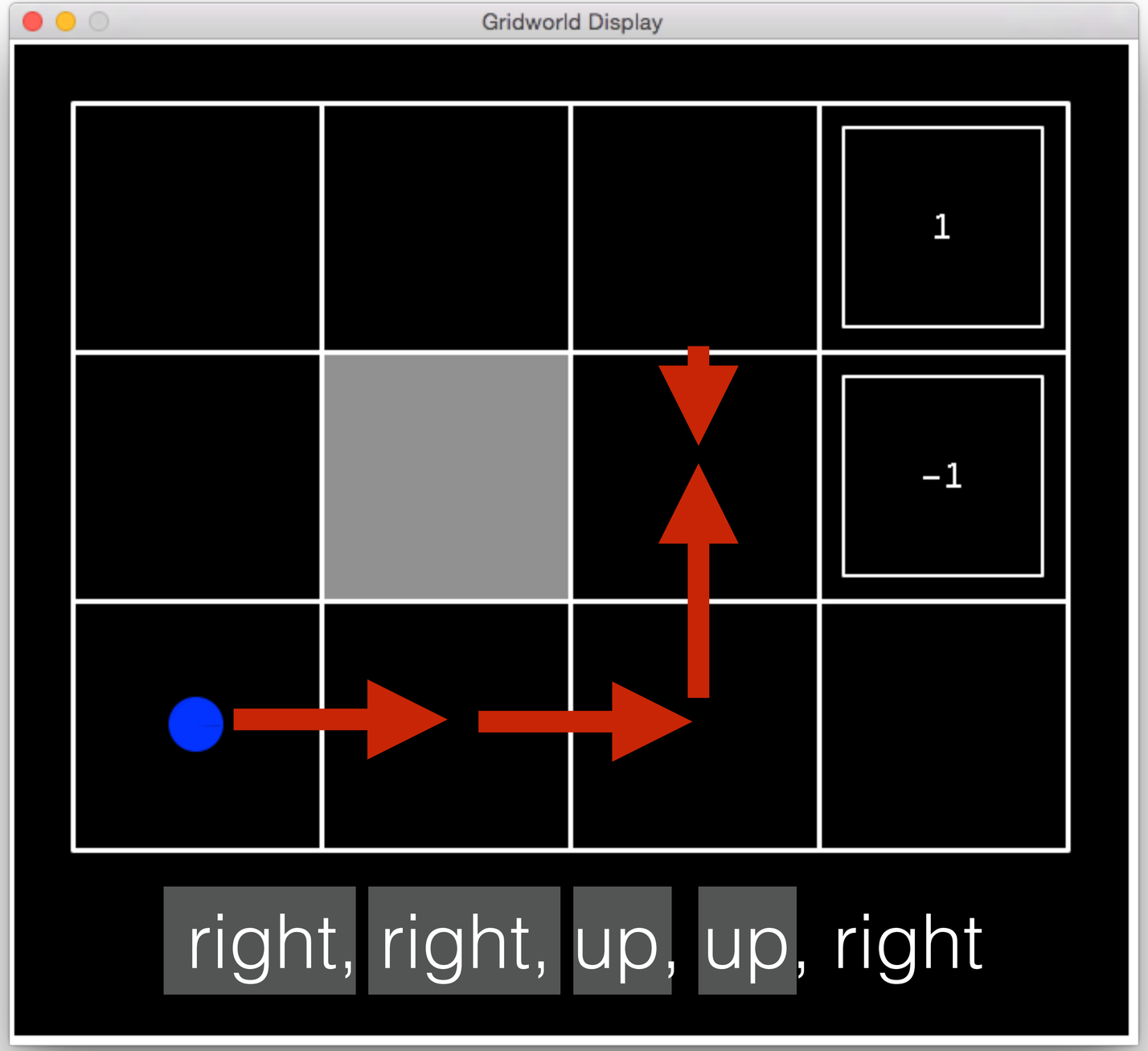

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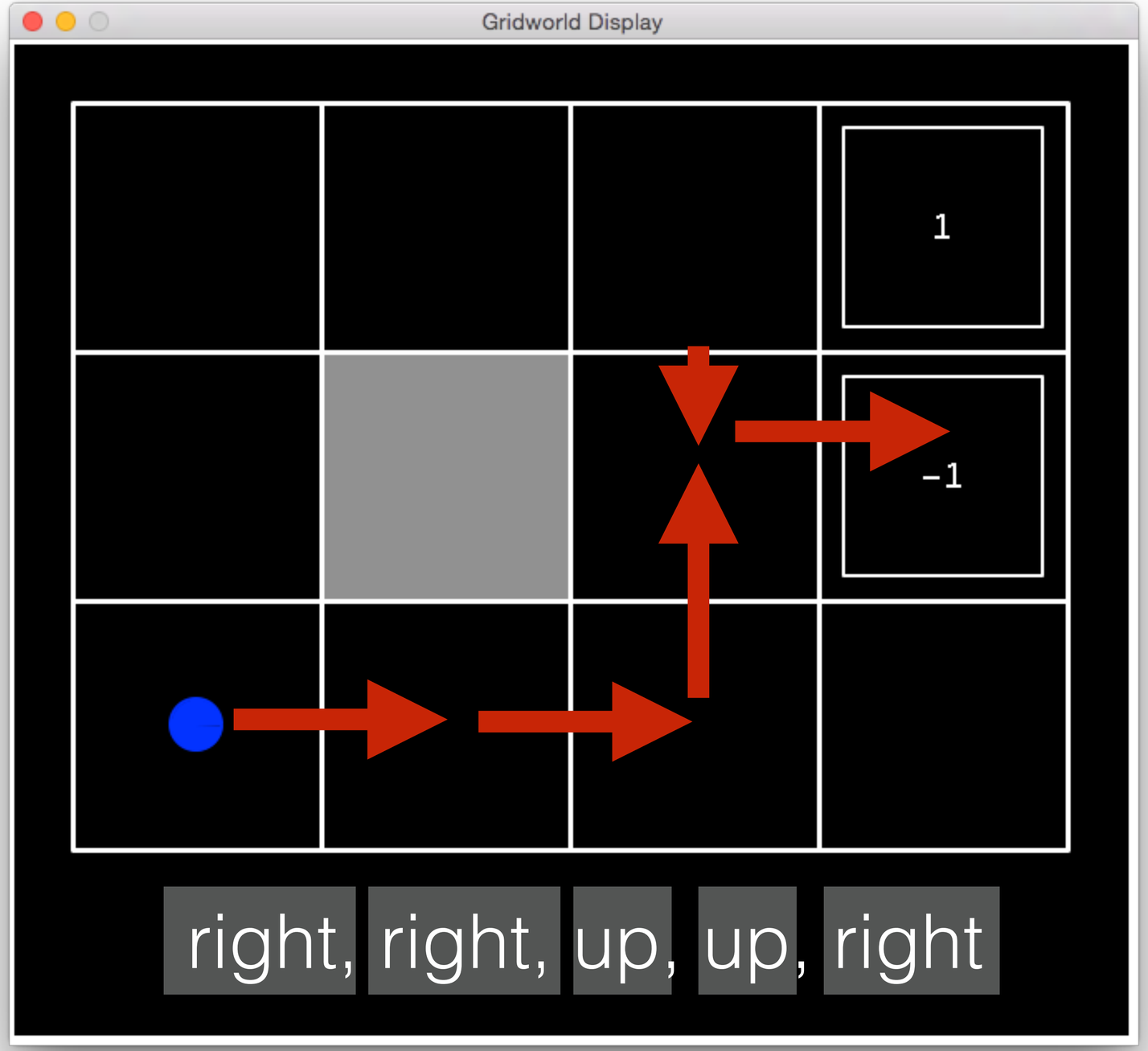

Actions and Transitions

- Pr(**s’** | **s**, **a**)
  - Probability we **transition** to **s’** if we choose **action a** in state **s**

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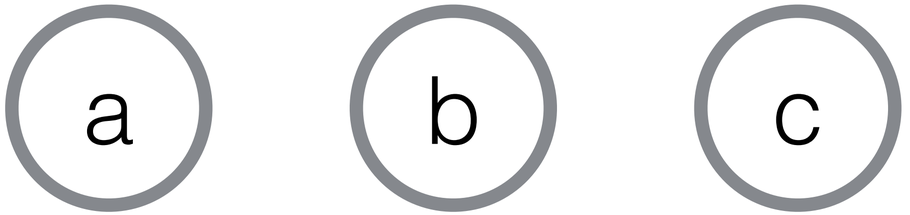

a = right

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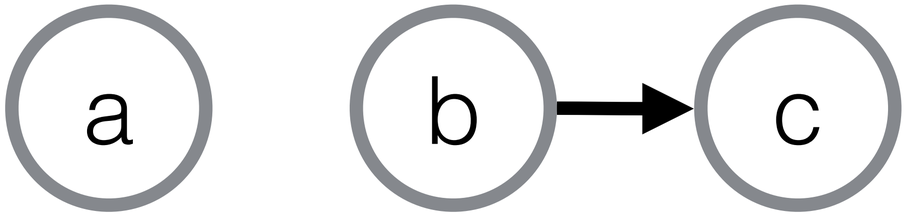

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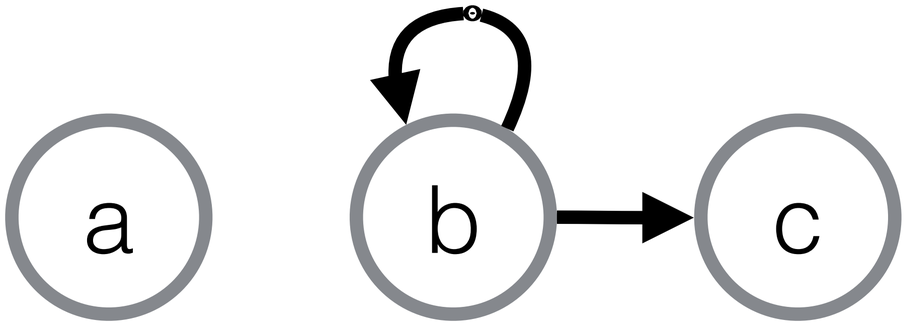

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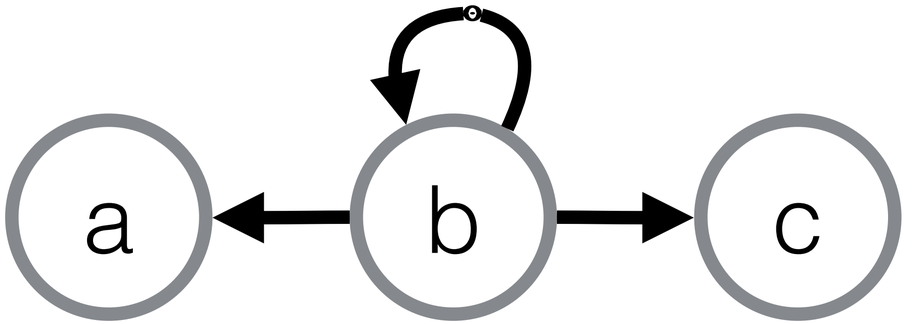

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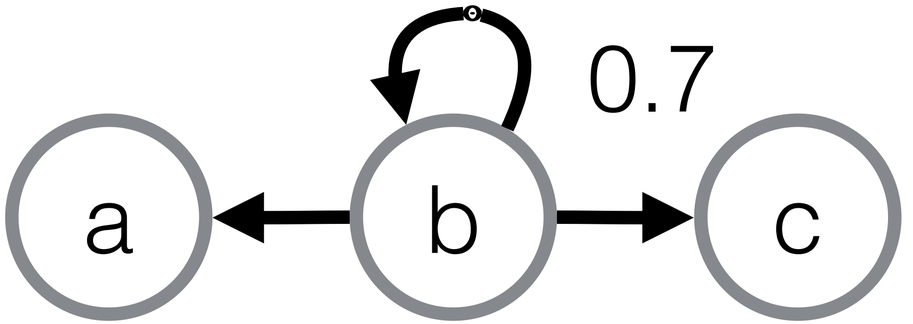

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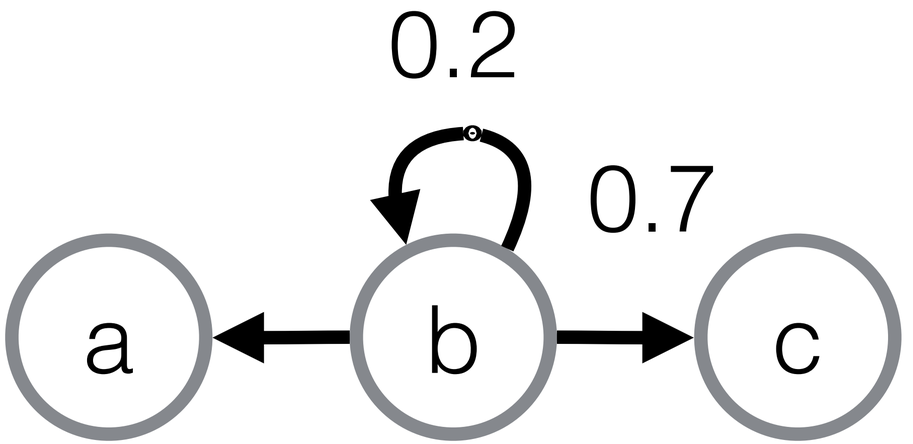

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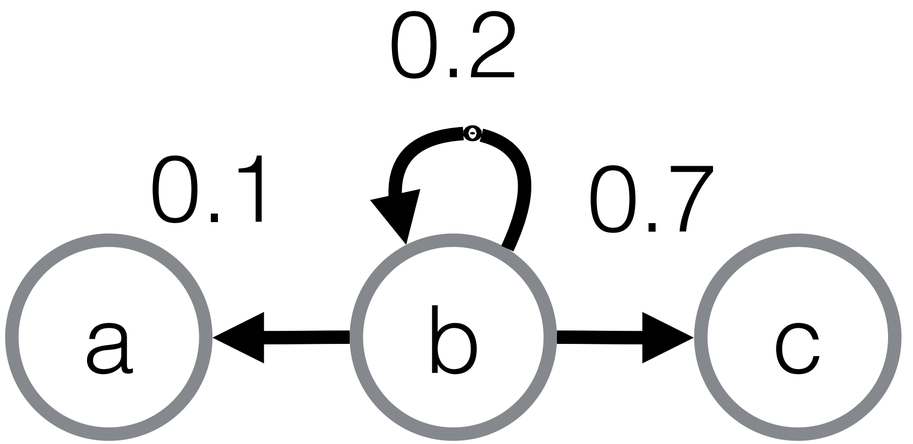

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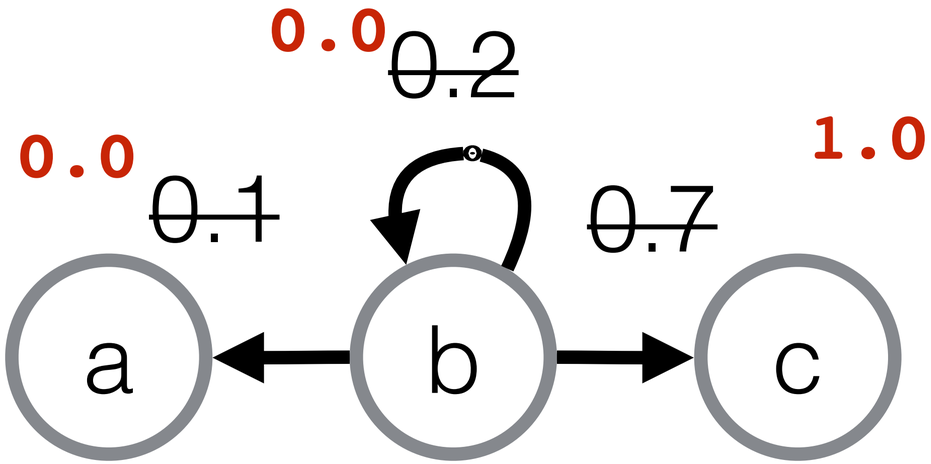

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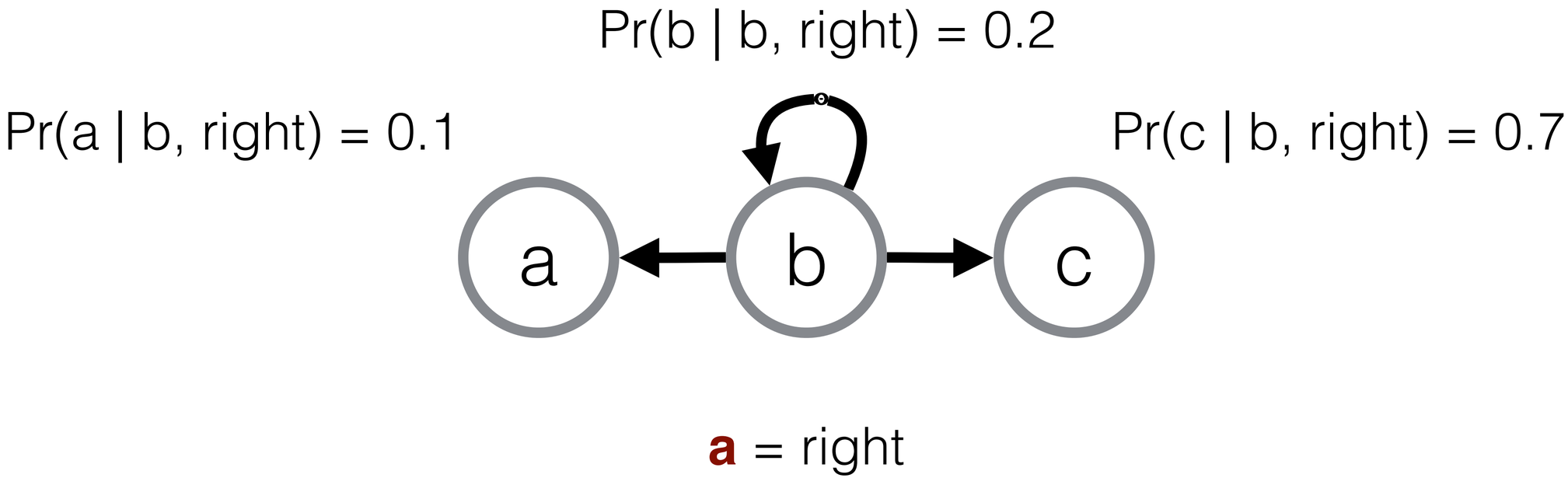

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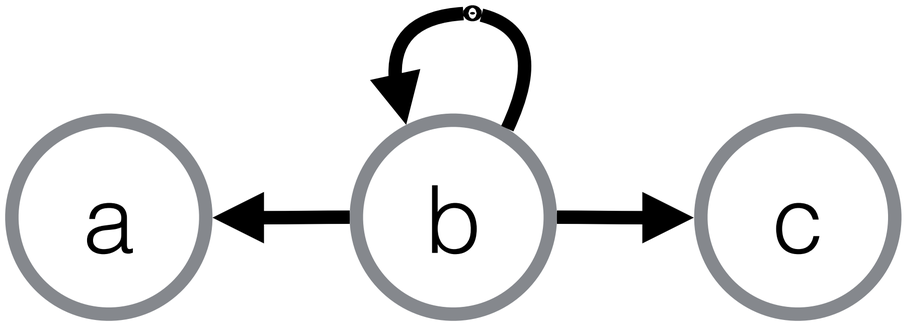

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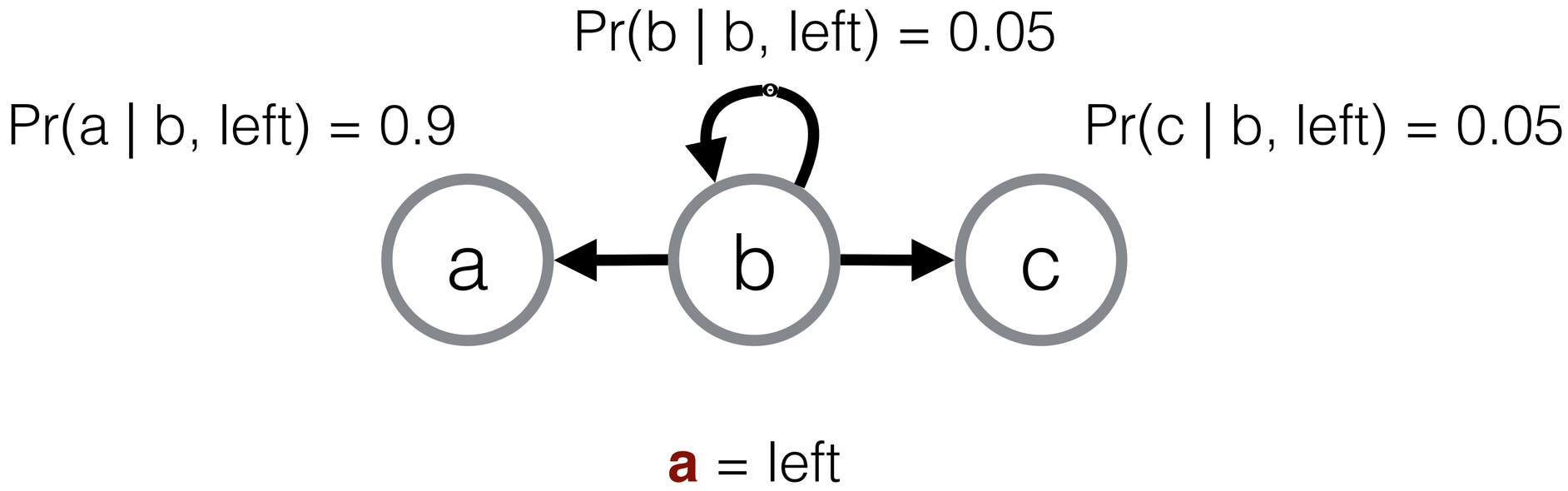

Preview: Markov Models

Markov Decision Process: Pr(**s’** | **s, a**)

Markov Process Pr(**s’** | **s**)

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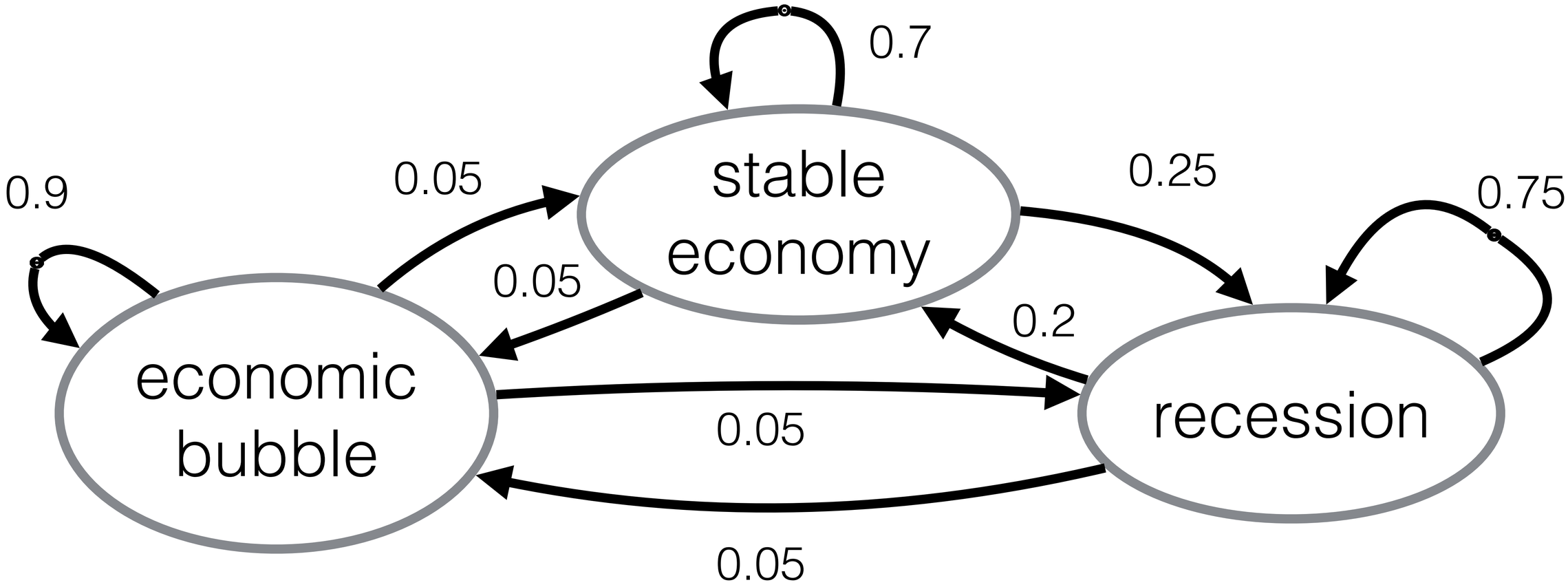
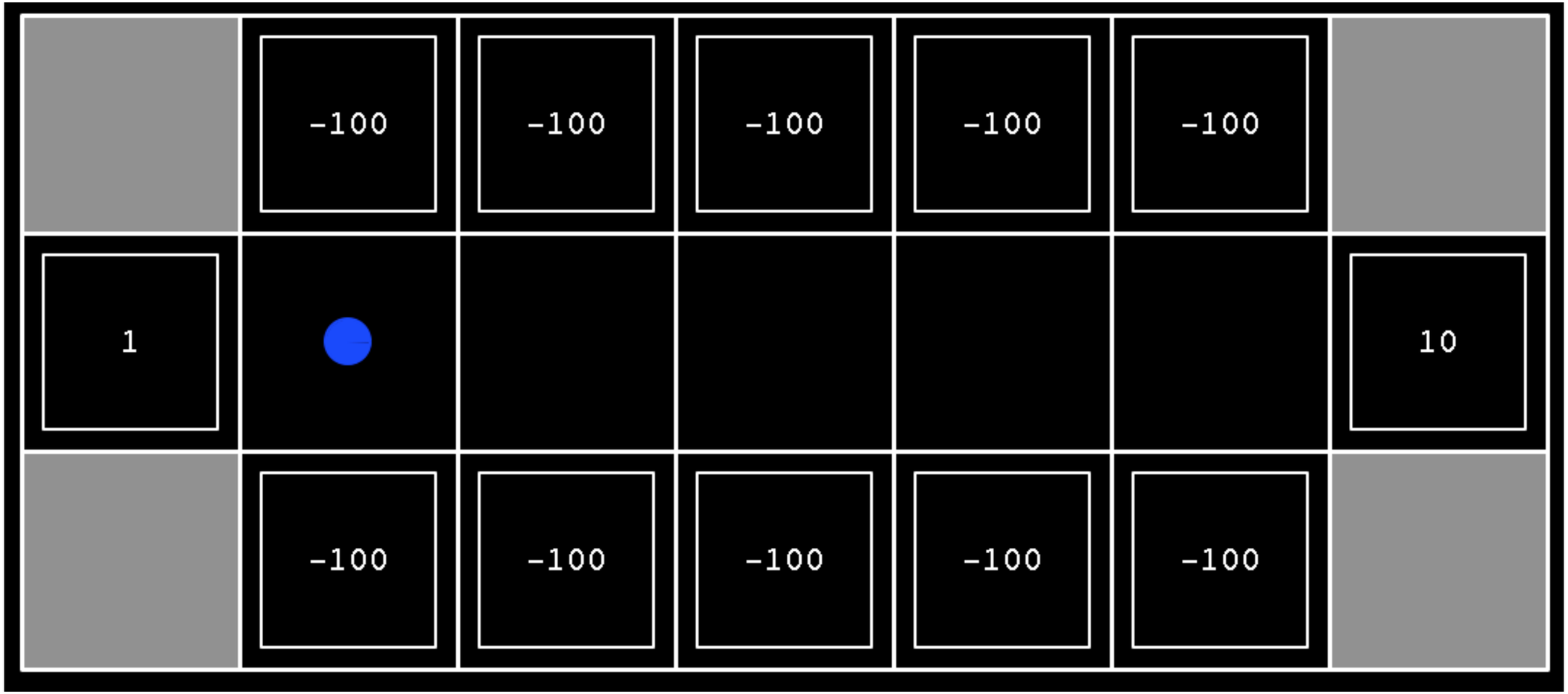
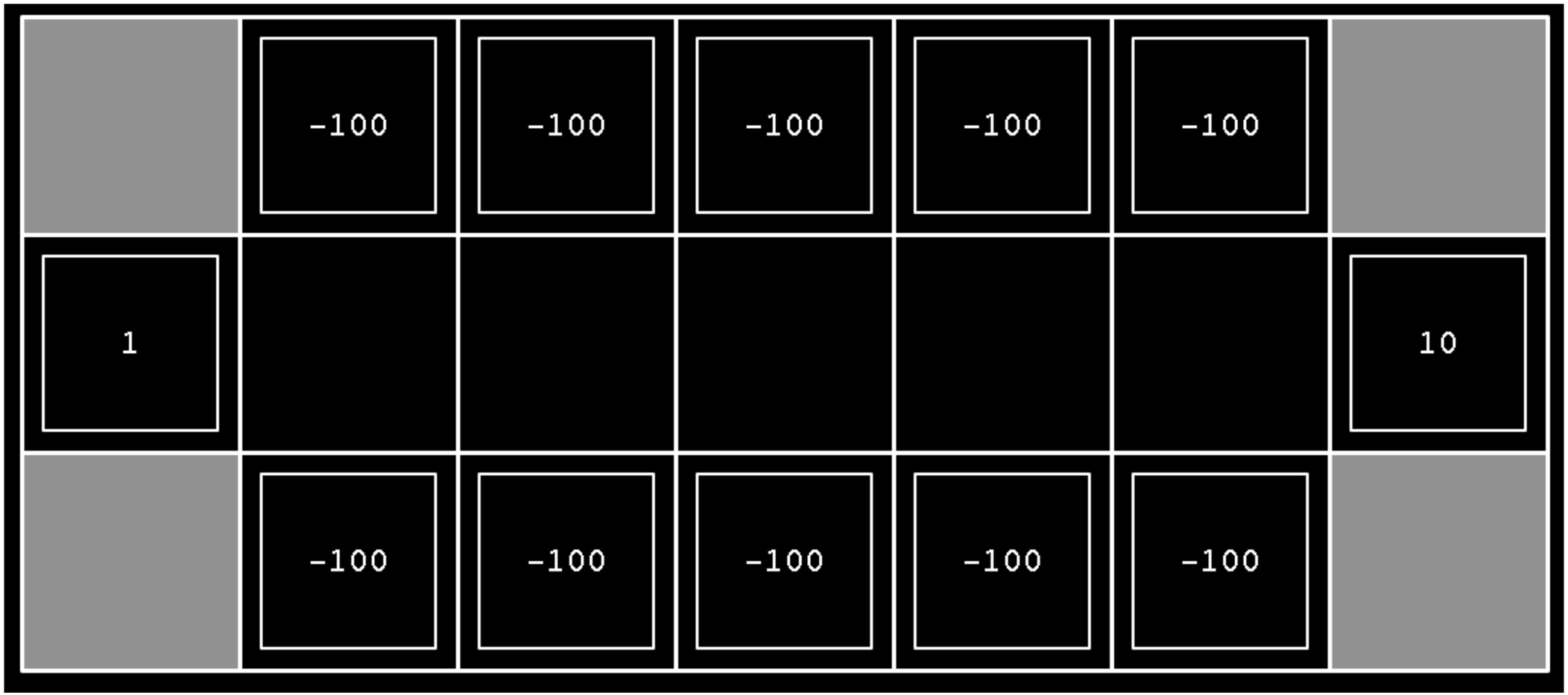


Markov Process Pr(**s'** | **s**)

Reward function R(s)



Policy $\pi(s)$



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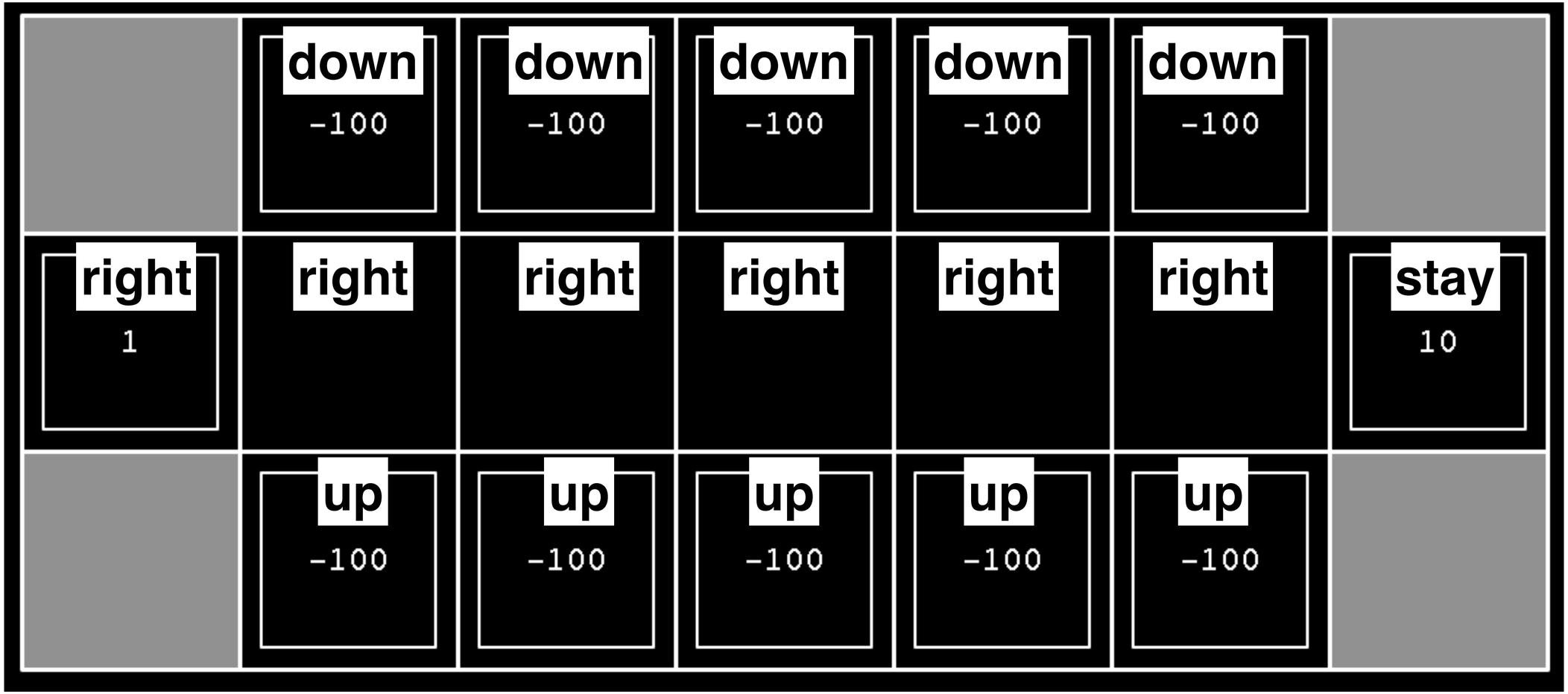
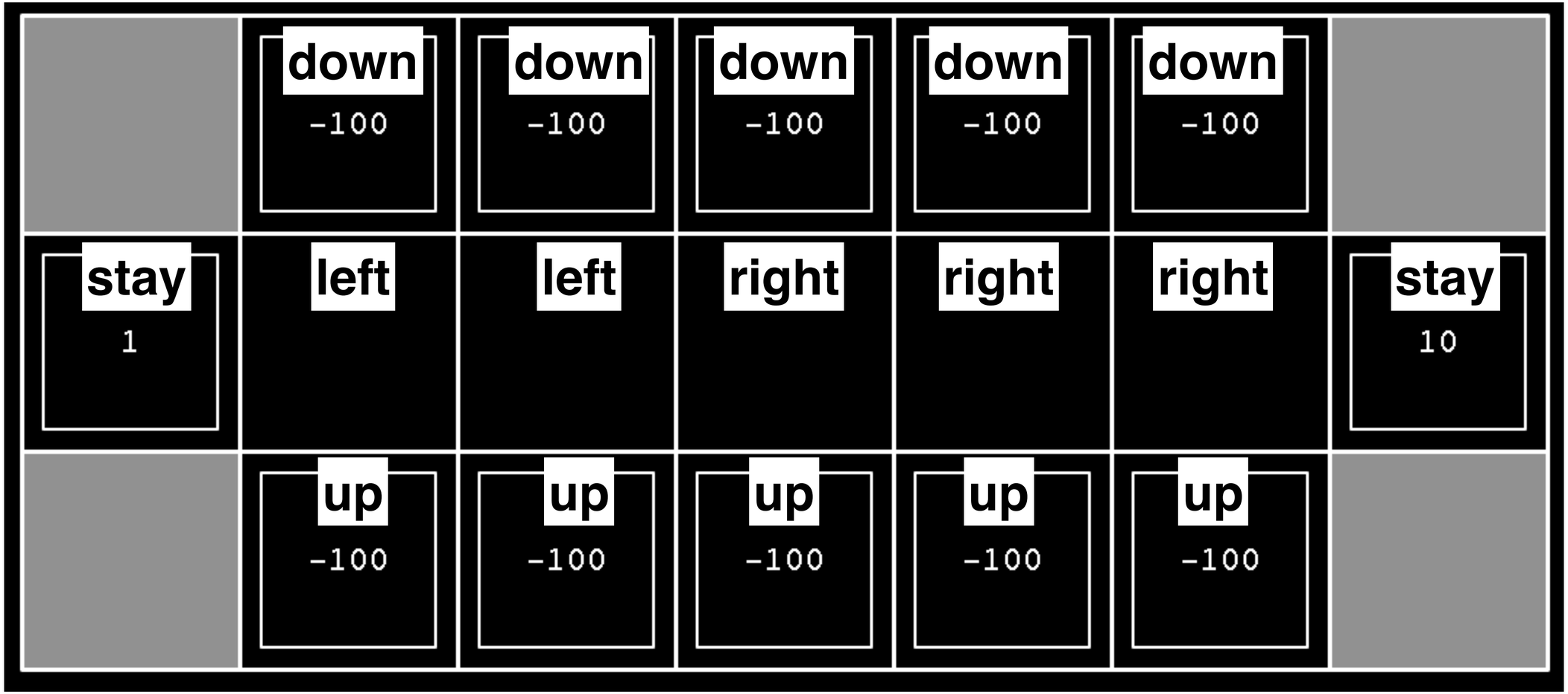

down
-100
down
-100
down
-100
down
-100
down
-100
right
1
right
right
right
right
right
stay
10
up
-100
up
-100
up
-100
up
-100
up
-100

Policy π(s)


down
-100
down
-100
down
-100
down
-100
down
-100
stay
1
left
left
right
right
right
stay
10
up
-100
up
-100
up
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up
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up
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How Good is a Policy?

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Bellman equation

Value Iteration

$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) U_i(s')$$

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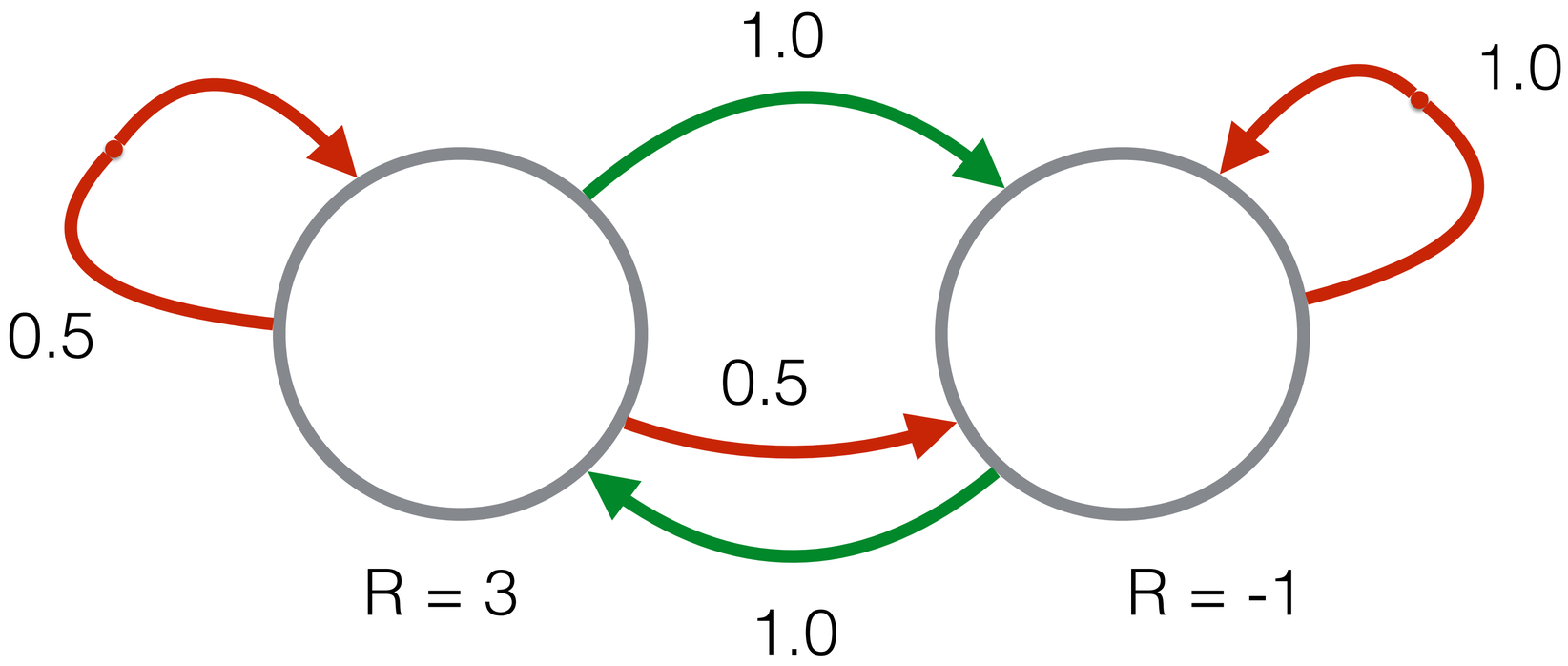
Value Iteration Example



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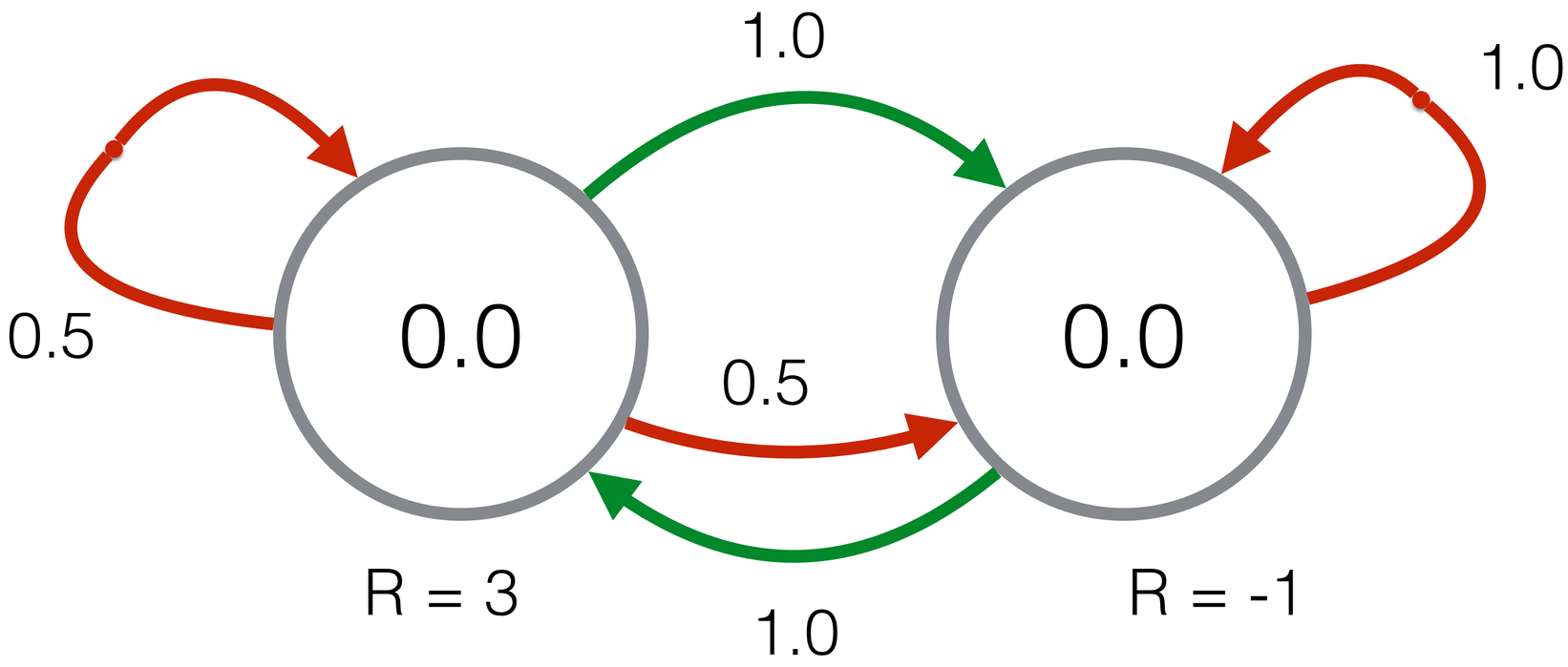
$$\gamma = 0.5$$



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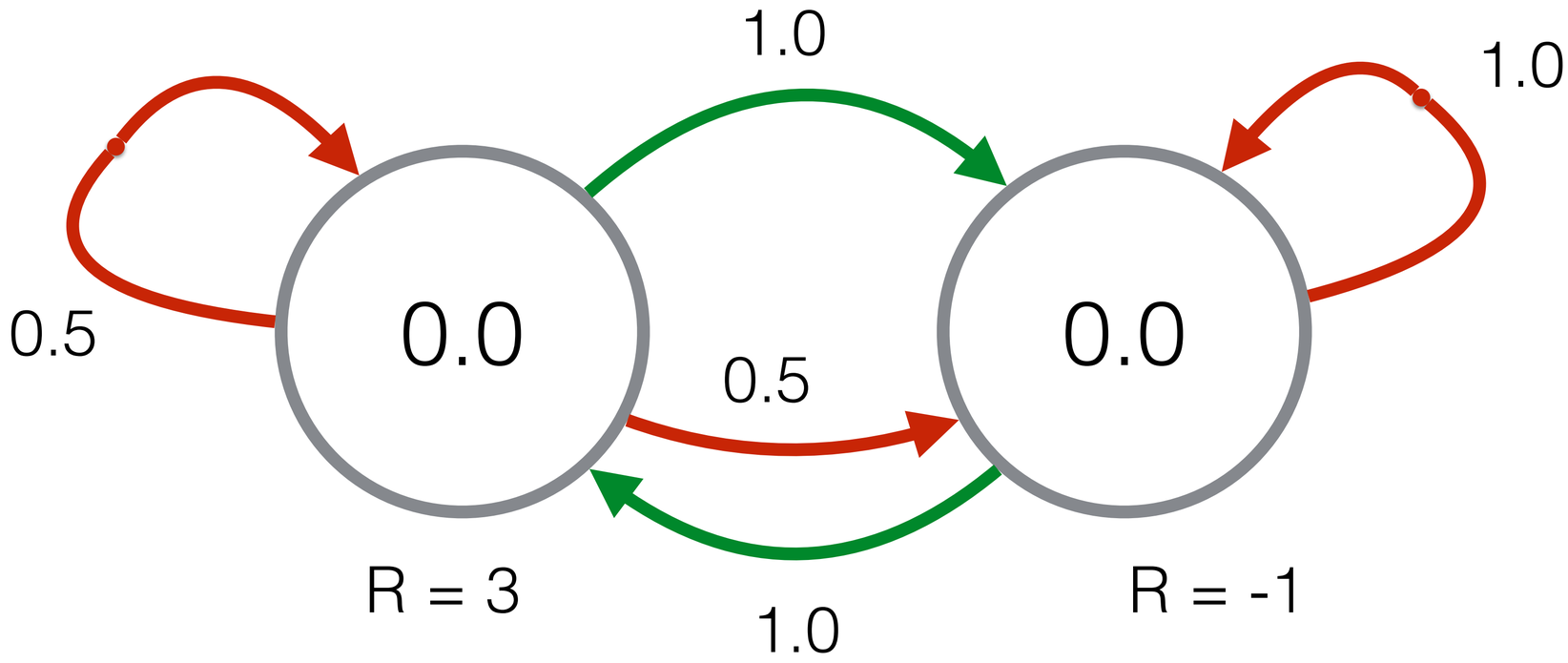

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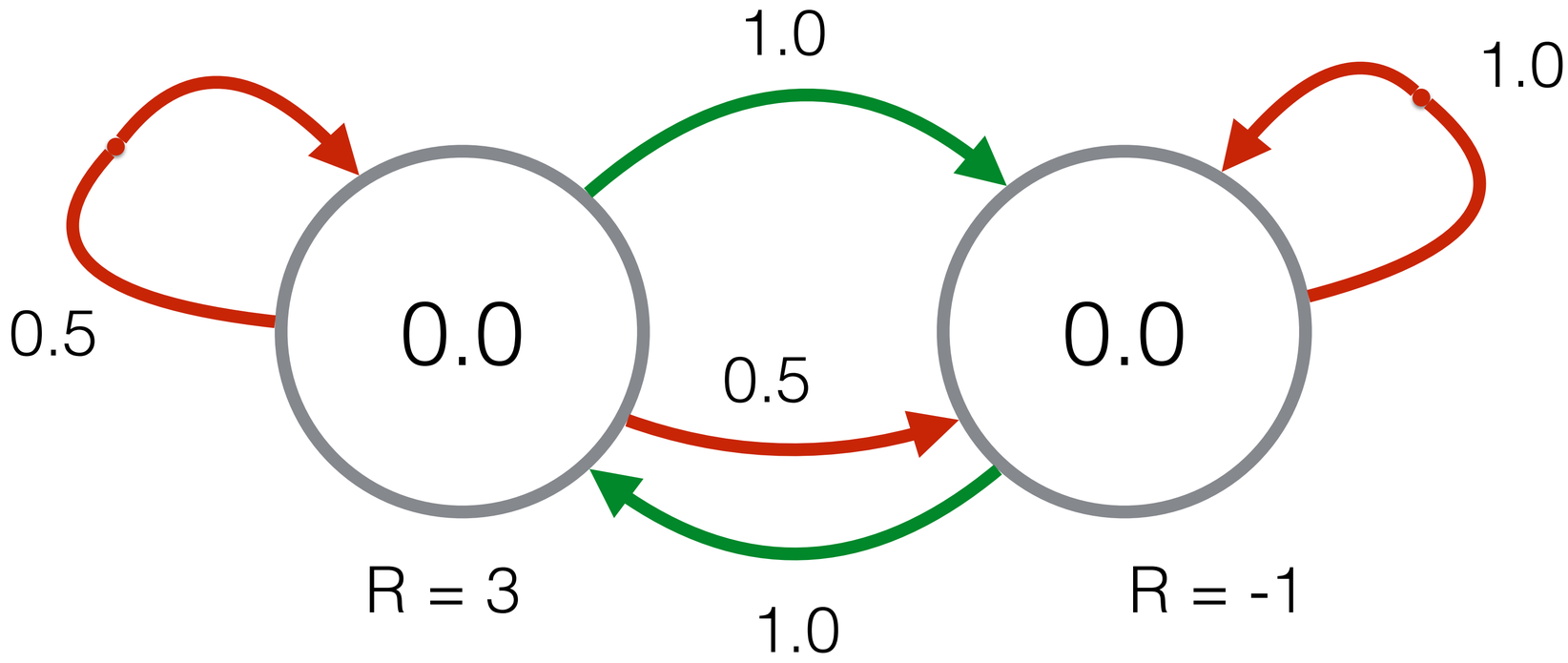
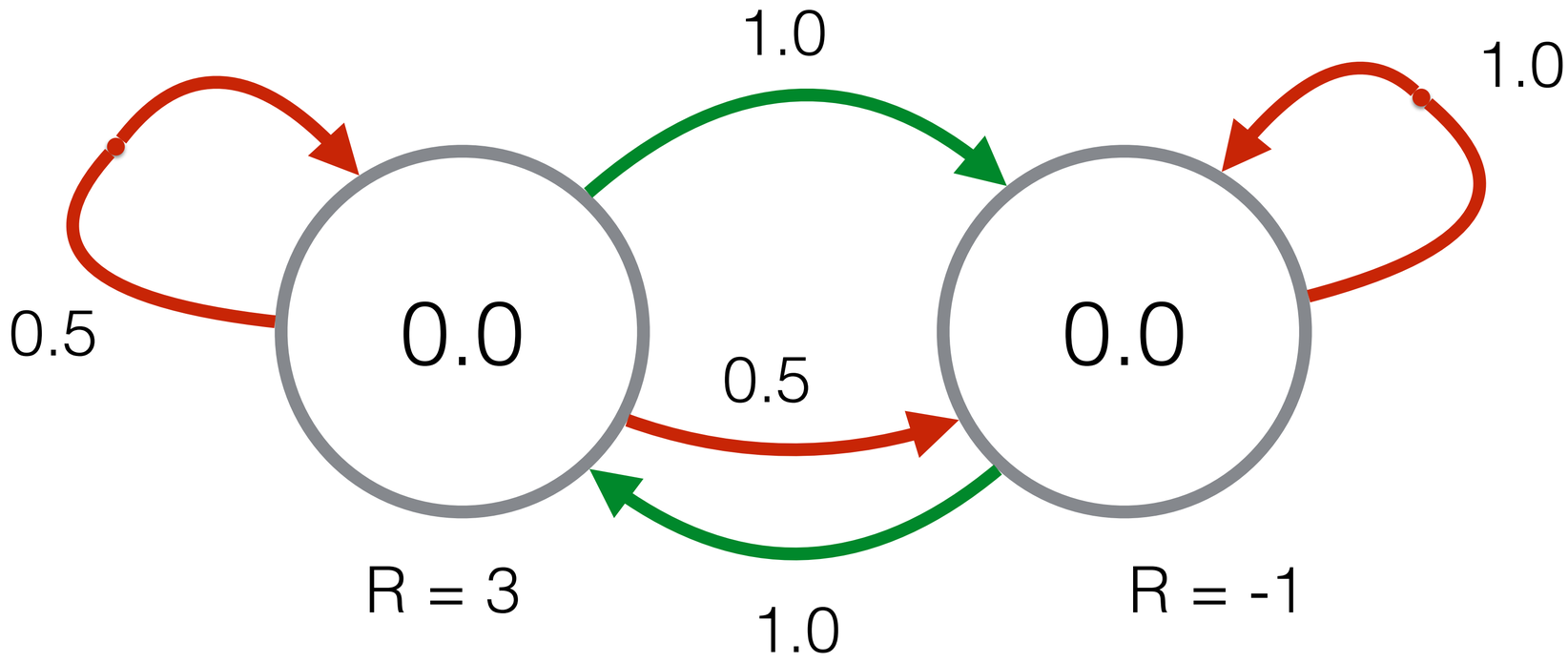


3 + 0.5 max{ 1.0 * 0.0, 0.5 * 0.0 + 0.5 * 0.0 } = 3

$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) U_i(s')$$

$$\gamma = 0.5$$



3 + 0.5 max{ 1.0 * 0.0, 0.5 * 0.0 + 0.5 * 0.0 } = 3

-1 + 0.5 max{ 1.0 * 0.0, 1.0 * 0.0 } = -1

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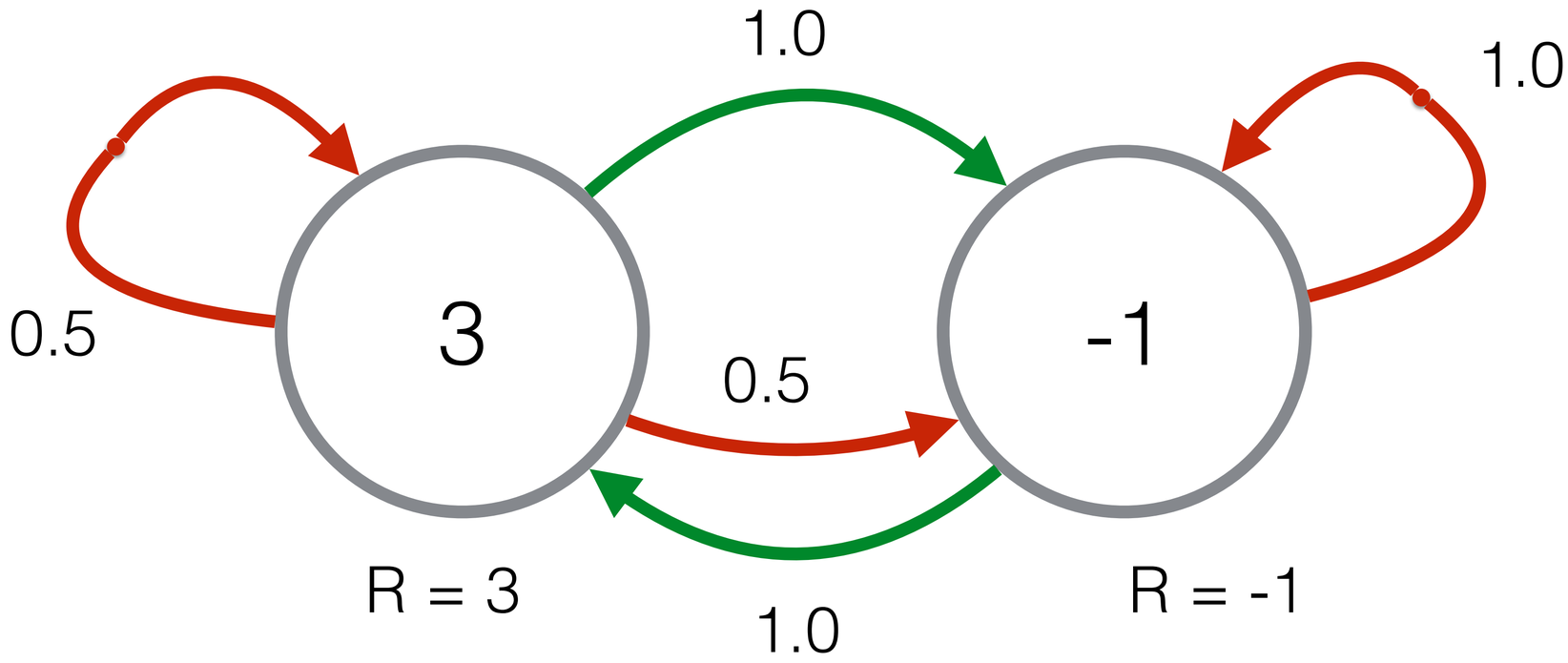
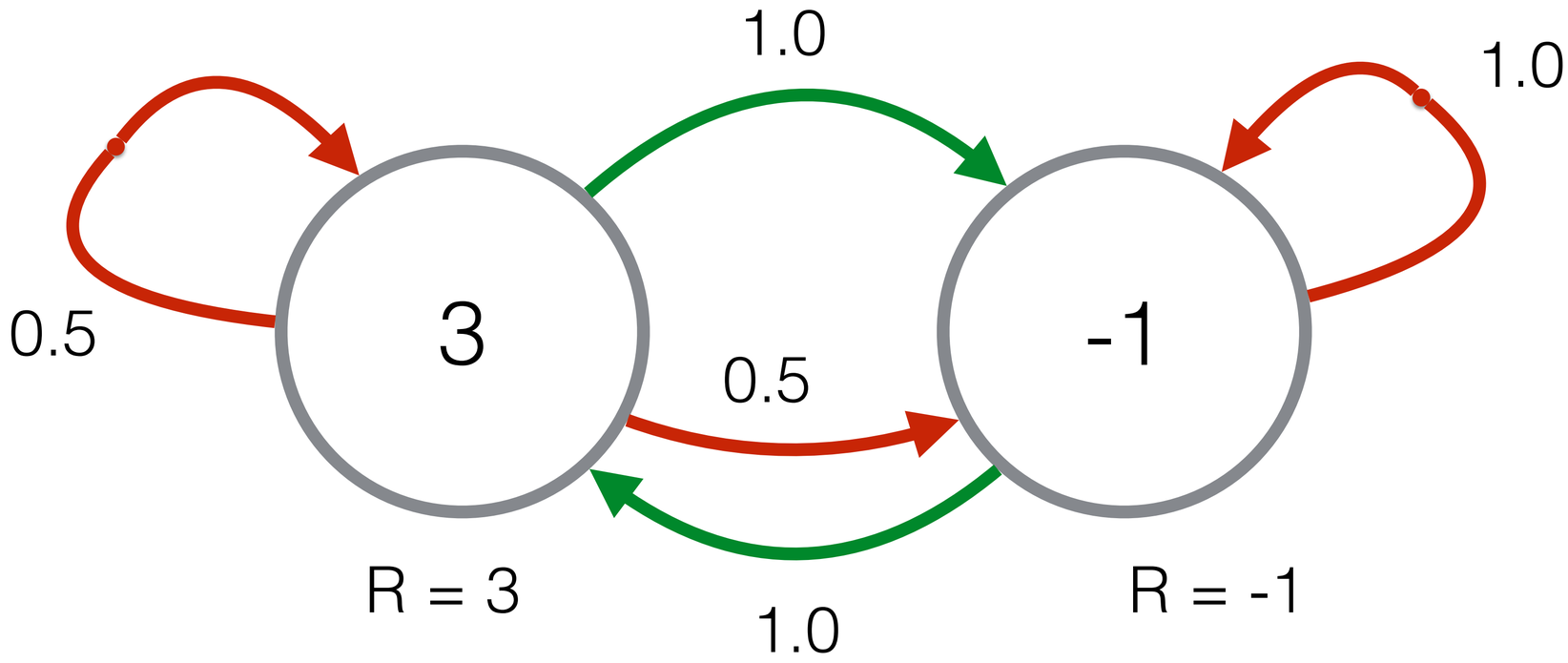
$$\gamma = 0.5$$



$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) U_i(s')$$

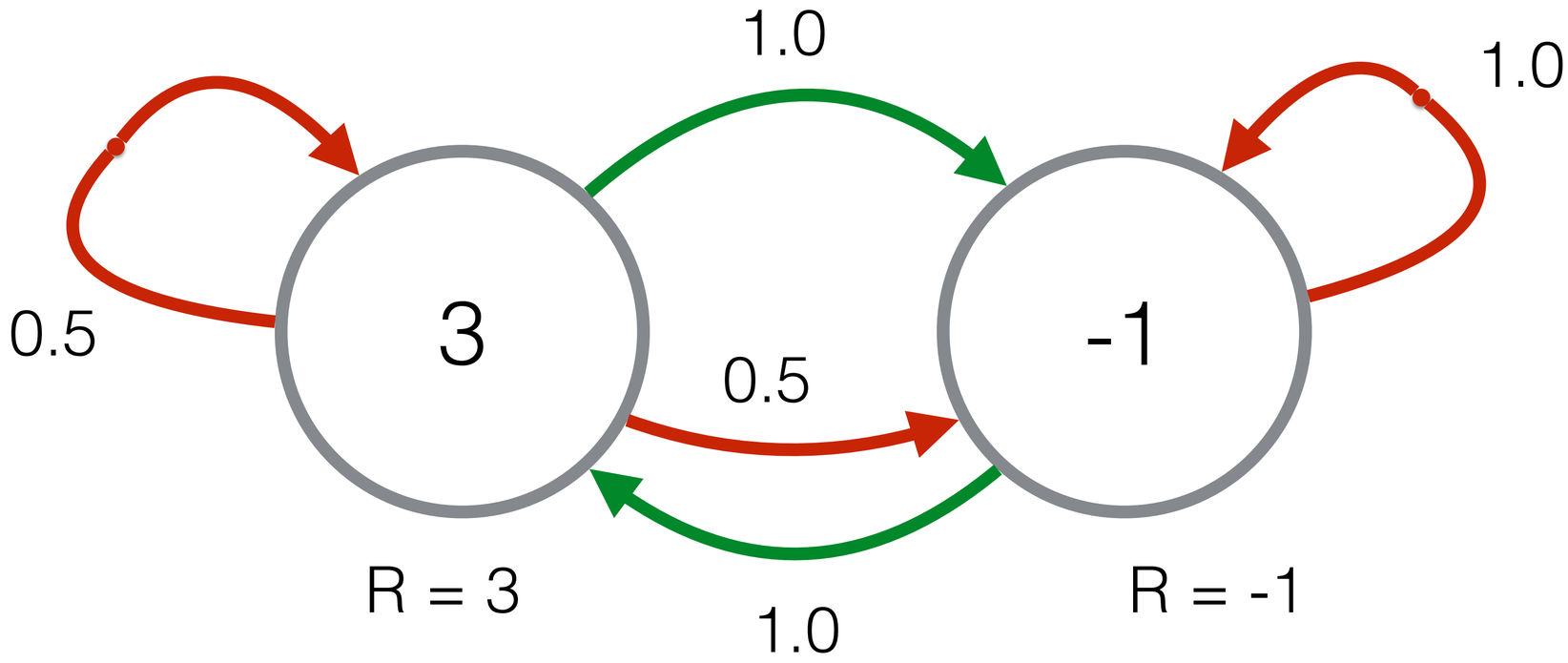
$$\gamma = 0.5$$



3 + 0.5 max{ 1.0 * (-1), 0.5 * 3 + 0.5 * (-1) } = 3 + 0.5 max{ -1, 1} = 3.5

$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) U_i(s')$$

$$\gamma = 0.5$$



3 + 0.5 max{ 1.0 * (-1), 0.5 * 3 + 0.5 * (-1) } = 3 + 0.5 max{ -1, 1} = 3.5

-1 + 0.5 max{ 1.0 * 3, 1.0 * (-1) } = 0.5

$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) U_i(s')$$

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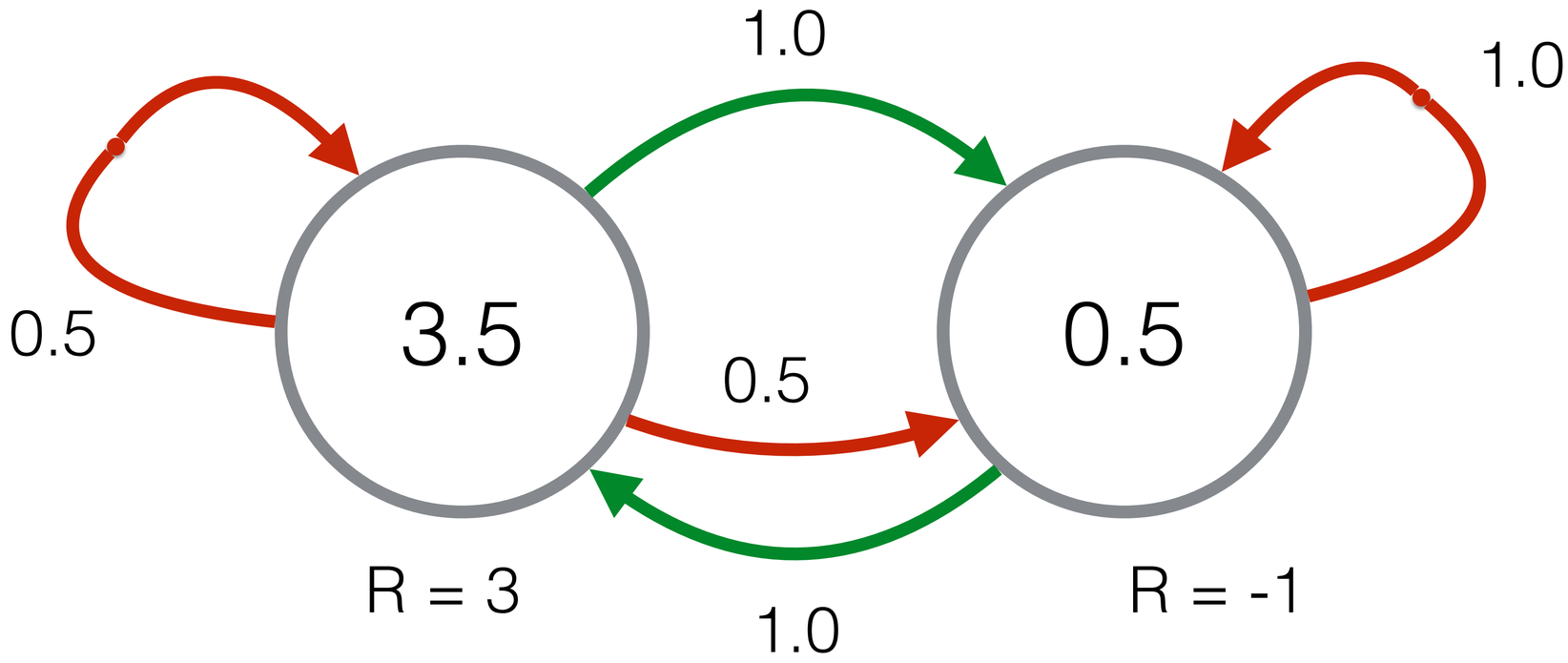

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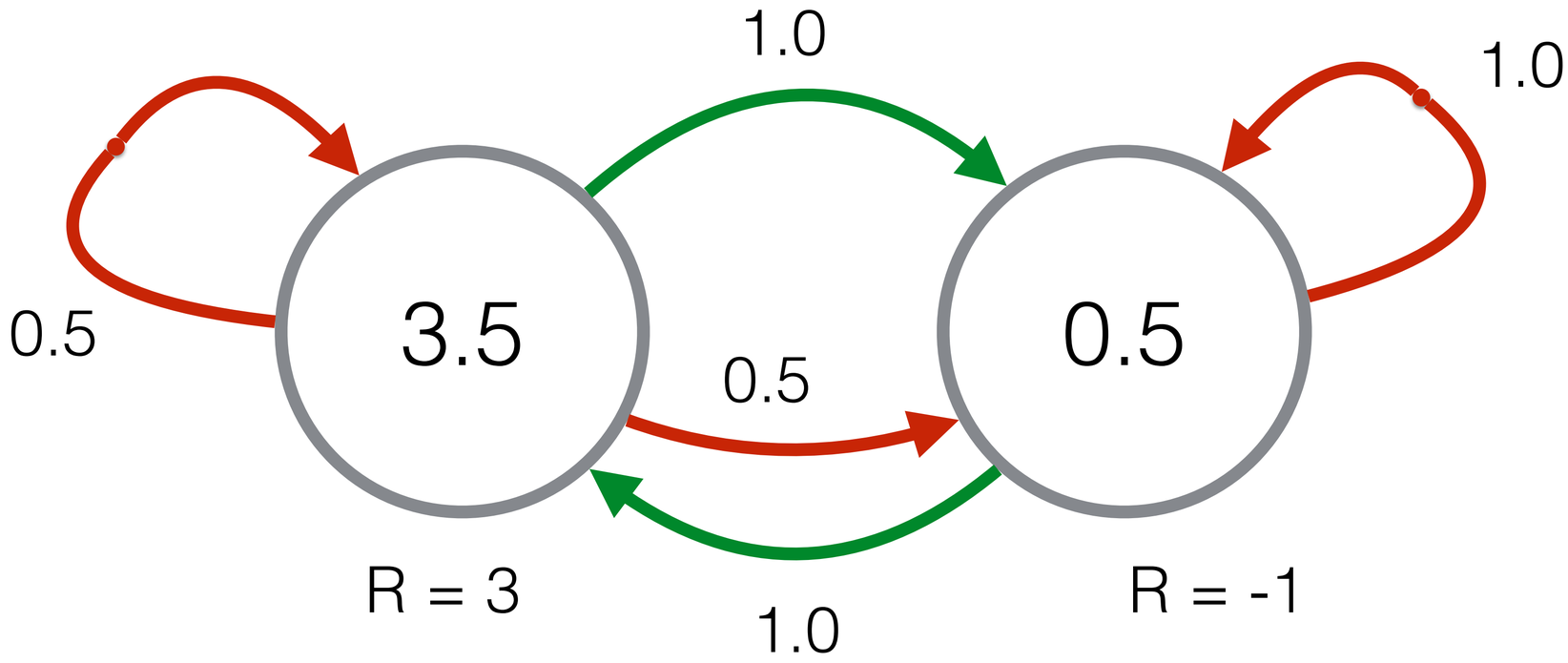
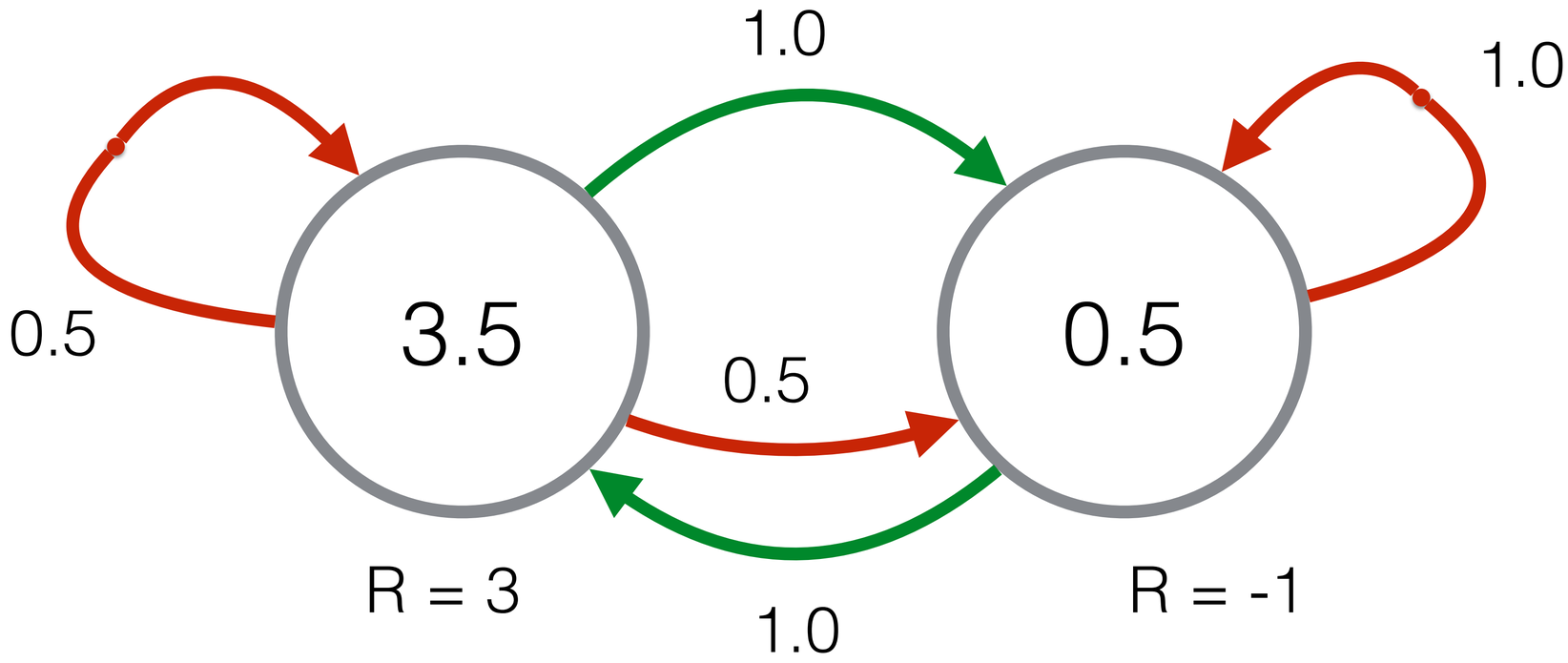


3 + 0.5 max{ 1.0 * 0.5, 0.5 * 3.5 + 0.5 * 0.5 } = 3 + 0.5 max{ 0.5, 2} = 4

$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) U_i(s')$$

$$\gamma = 0.5$$

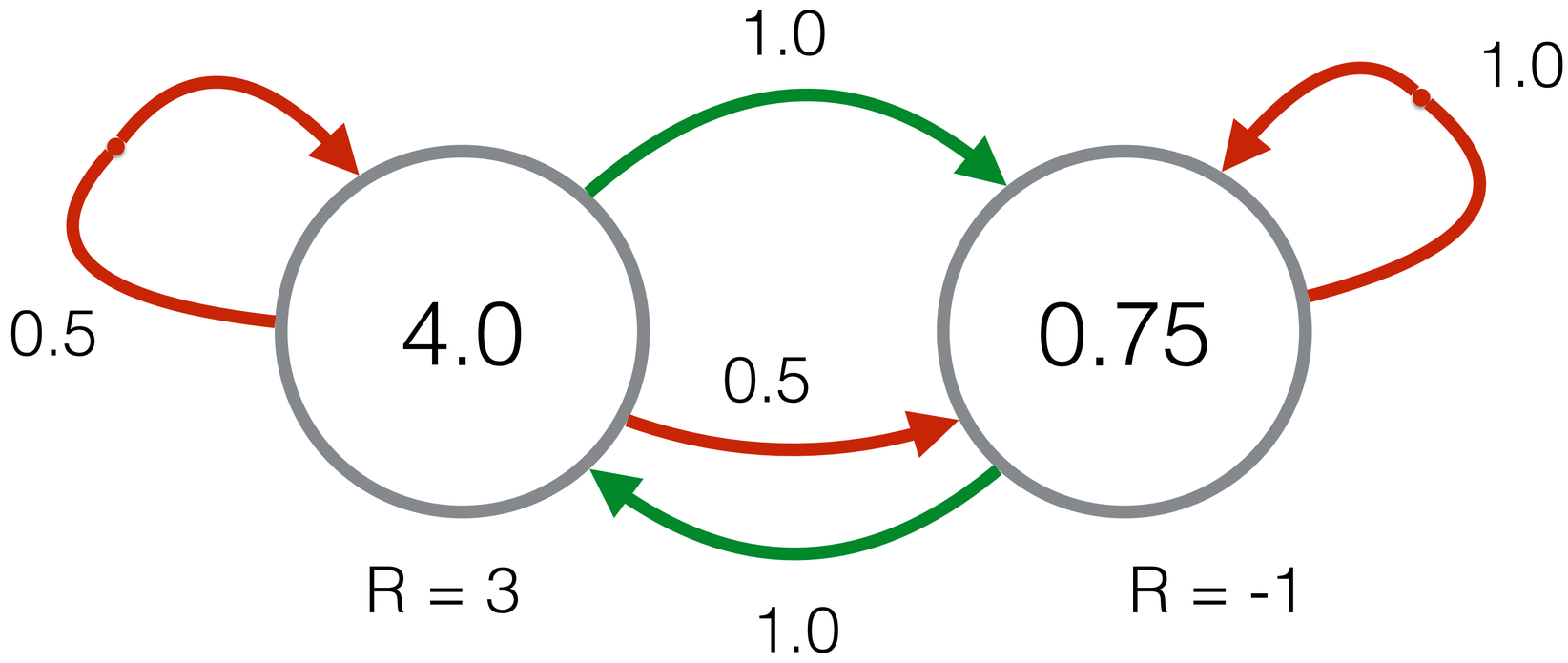


3 + 0.5 max{ 1.0 * 0.5, 0.5 * 3.5 + 0.5 * 0.5 } = 3 + 0.5 max{ 0.5, 2} = 4

-1 + 0.5 max{ 1.0 * 3.5, 1.0 * 0.5 } = 0.75

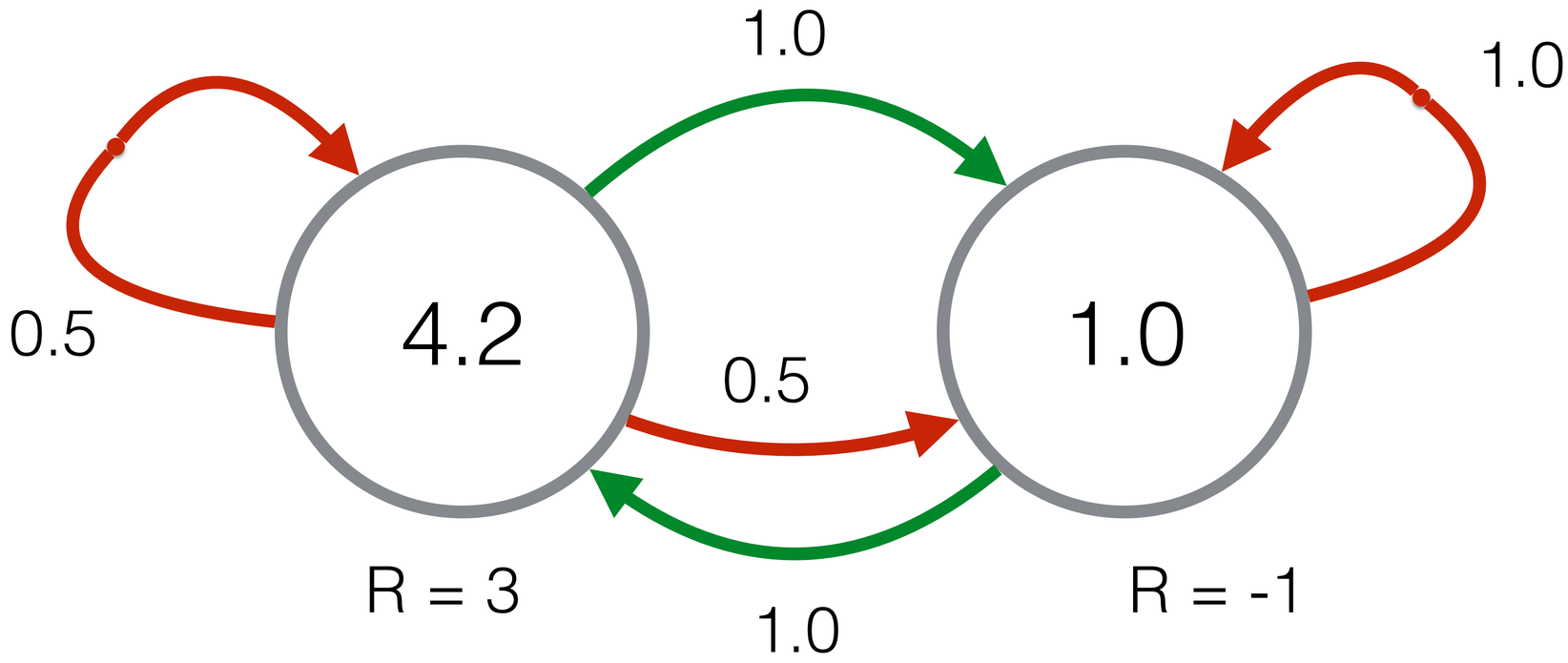
$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) U_i(s')$$

$$\gamma = 0.5$$



$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) U_i(s')$$

$$\gamma = 0.5$$



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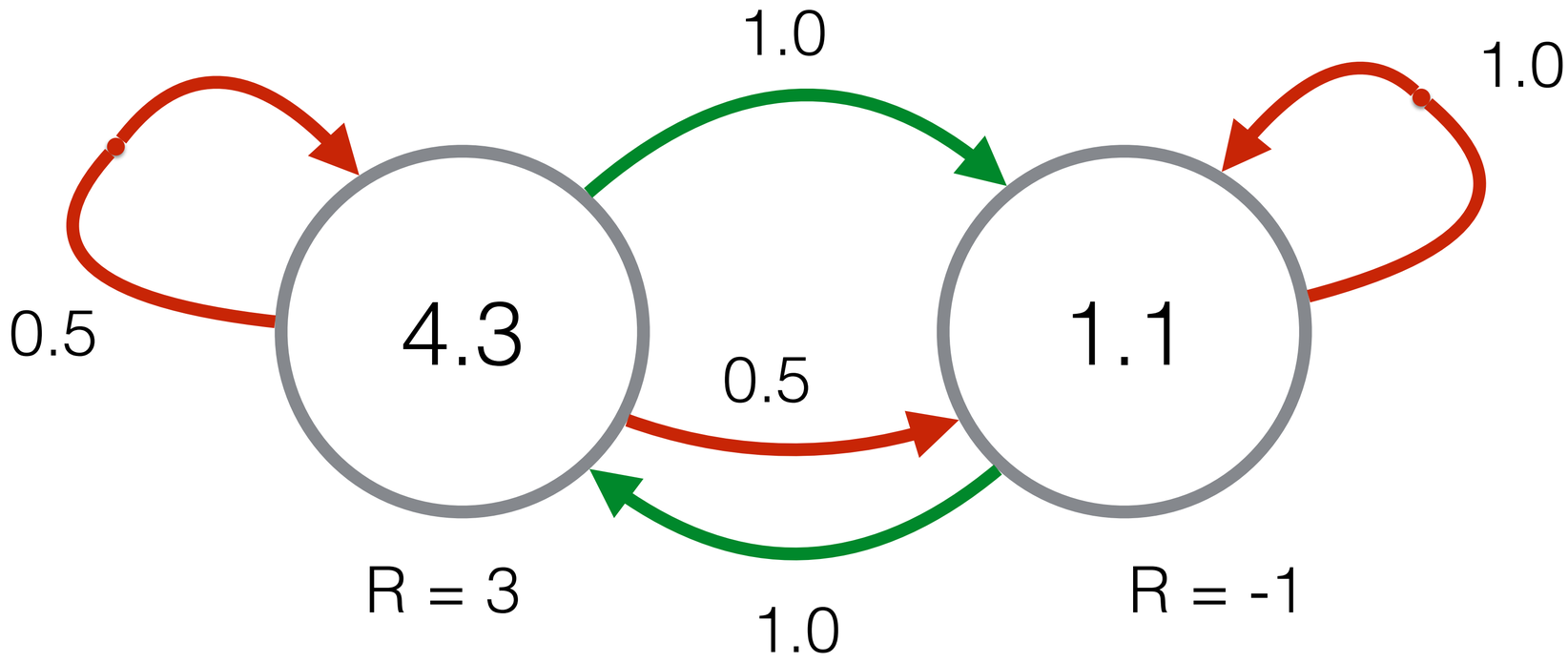

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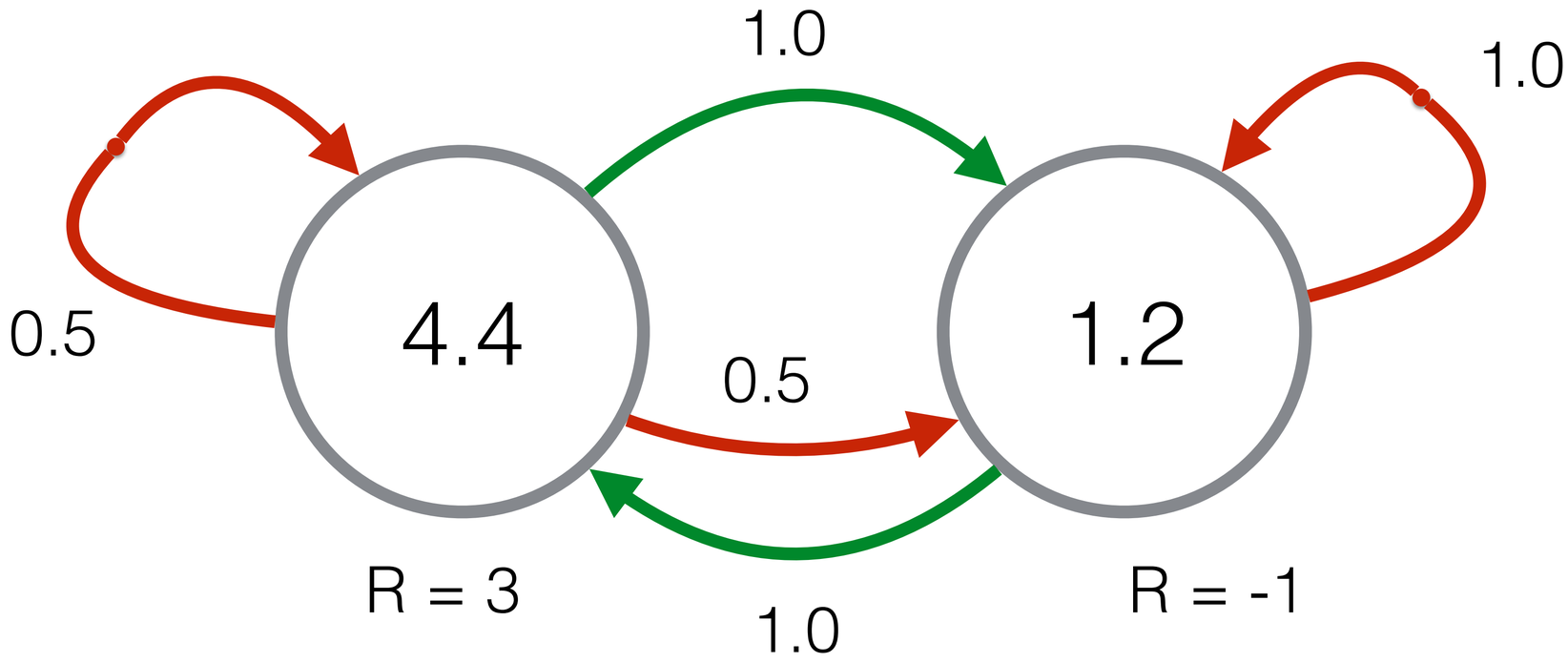

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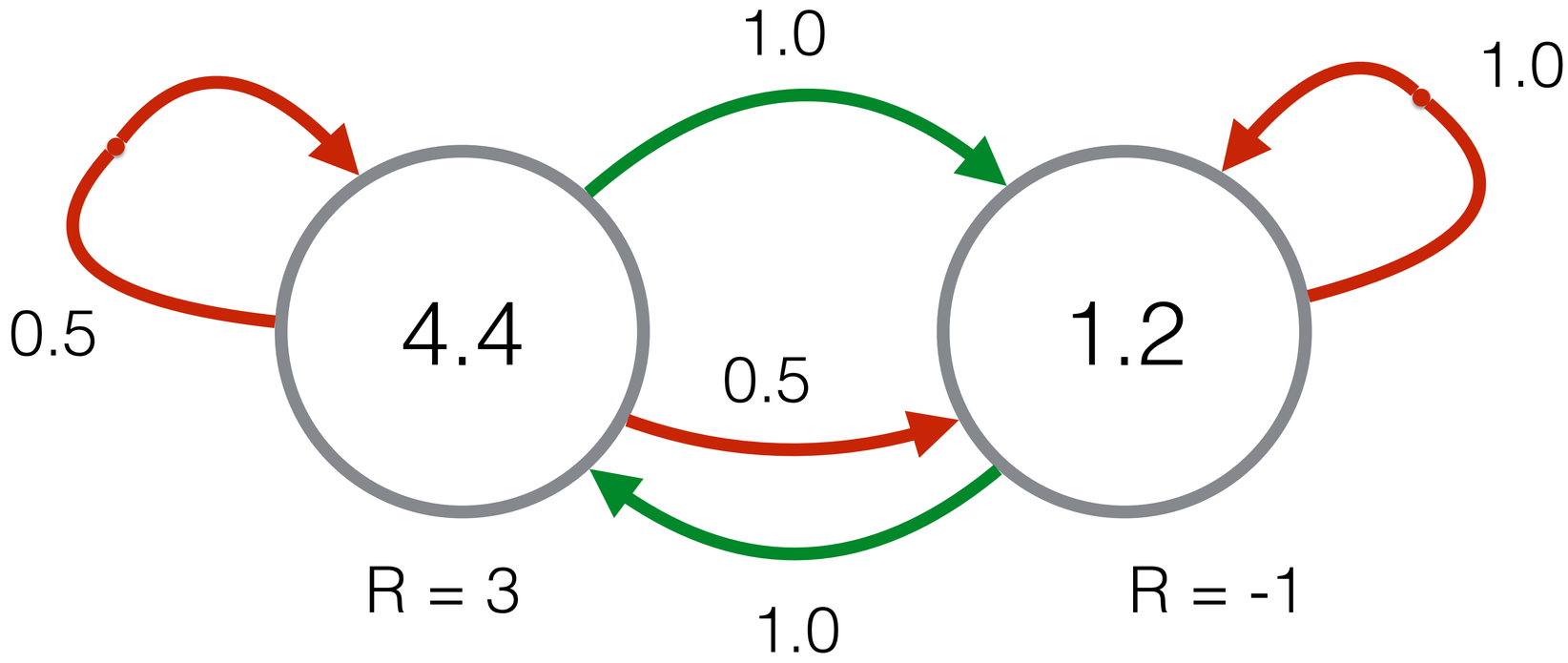

3 + 0.5 max { 1.0 * 1.2, 0.5 * 4.4 + 0.5 * 1.2 }

$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) U_i(s')$$

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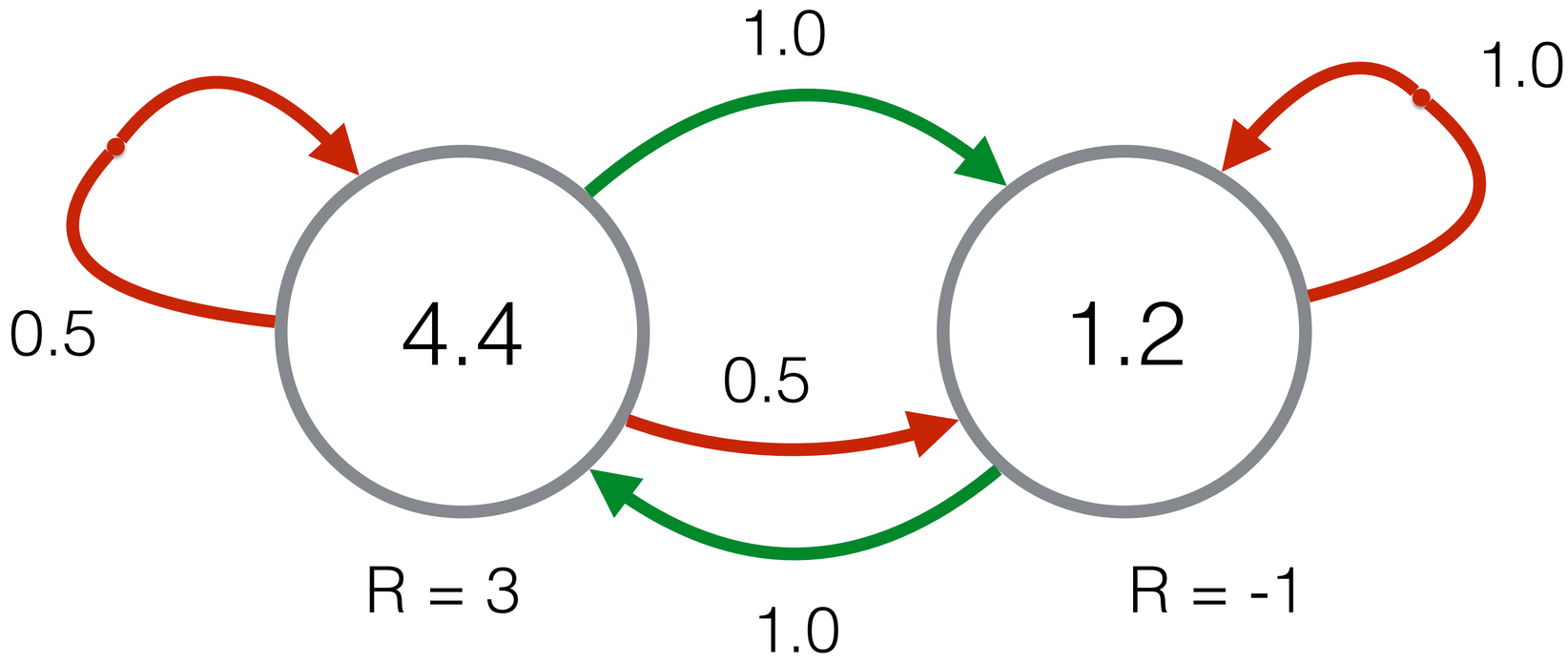
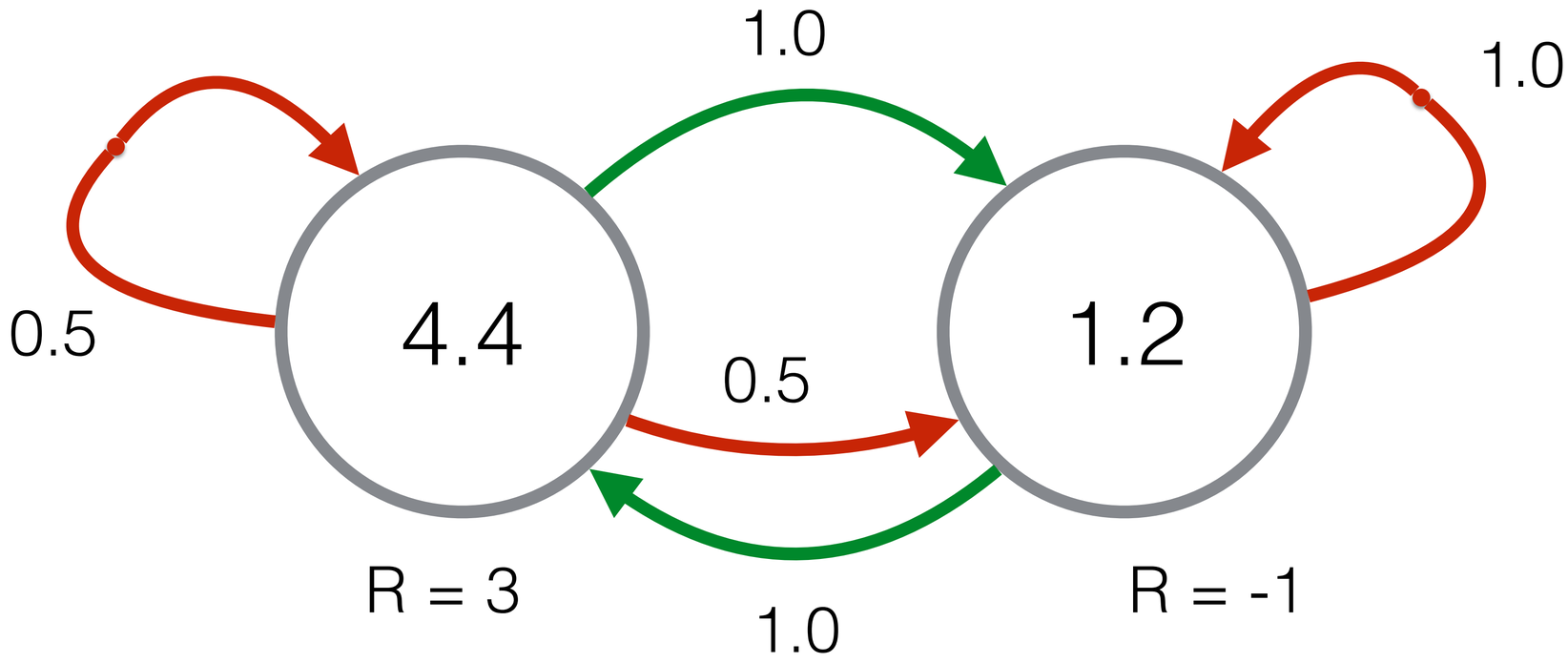


3 + 0.5 max { 1.0 * 1.2, 0.5 * 4.4 + 0.5 * 1.2 }

3 + 0.5 max { 1.2, 2.2 + 0.6 }

$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) U_i(s')$$

$$\gamma = 0.5$$



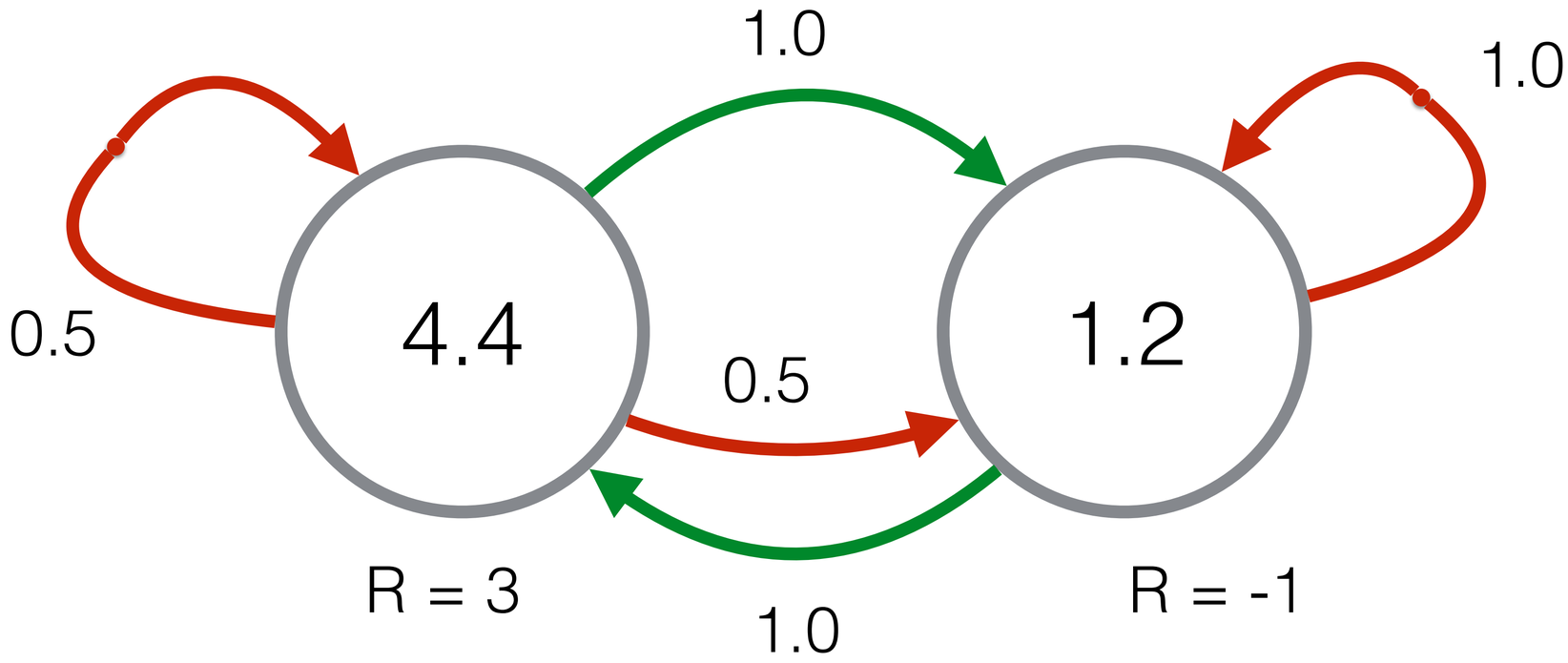
3 + 0.5 max { 1.0 * 1.2, 0.5 * 4.4 + 0.5 * 1.2 }

3 + 0.5 max { 1.2, 2.2 + 0.6 }

3 + 0.5 max { 1.2, 2.8 }

$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s'|s,a) U_i(s')$$

$$\gamma = 0.5$$



3 + 0.5 max { 1.0 * 1.2, 0.5 * 4.4 + 0.5 * 1.2 }

3 + 0.5 max { 1.2, 2.2 + 0.6 }

3 + 0.5 max { 1.2, 2.8 }

3 + 0.5 * 2.8 = 4.4

Summary

Summary

- Markov decision processes

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  - actions have probabilistic state transitions

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- Chapter 17 to end of 17.2