Logic

Virginia Tech CS 5804
Spring 2015

Outline

- Propositional logic syntax and semantics
- Inference in propositional logic
- (Horn clauses, forward/backward chaining)

Terminology

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  - and it can be **complete**: able to infer any sentence that is entailed

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  - **not, negation**: ¬X, ¬Y — !X
  - **and, conjunction**: X ∧ Y — X && Y
  - **or, disjunction**: X ∨ Y — X || Y
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literal: X, ¬X, Y, ¬Y, Z, ¬Z

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Inference

- Given KB with prop. logic sentences, check if sentence α is true
- Naive approach: evaluate full truth table
    - complete and sound inference but *expensive*
- Better approach: execute **inference rules** on KB: **theorem proving**

KB: X ^ Y	**X = F**	**X = T**
Y = F	F	F
Y = T	F	T

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Category:Rules of inference

From Wikipedia, the free encyclopedia

*The main article for this category is **Rules of inference**.*

The concepts described in articles in this category may be also expressed in terms of arguments, or theorems. Very often the same concept is in more than one of these categories, expressed a different way and sometimes with a different name.

Pages in category "Rules of inference"

The following 40 pages are in this category, out of 40 total. This list may not reflect recent changes (learn more).

- Rule of inference

*****

- List of rules of inference

A

- Absorption (logic)
- Admissible rule
- Associative property

D cont.

- Disjunction elimination
- Disjunction introduction
- Disjunctive syllogism
- Distributive property
- Double negative elimination

E

- Existential generalization
- Existential instantiation

M cont.

- Modus ponens
- Modus tollens

N

- Negation as failure
- Negation introduction

R

- Resolution (logic)

Inference via Search

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- States: knowledge bases
  Actions: apply inference rule, add new sentence to KB
  Goal: add target sentence α to KB

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- Sound? Yes!

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Inference via Search

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  Goal: add target sentence α to KB
- Sound? Yes!
- Complete? Not necessarily

Resolution

$$\frac{X_1 \lor X_2 \lor \ldots \lor X_N \lor \beta, \quad Y_1 \lor Y_2 \lor \ldots \lor Y_M \lor \neg\beta}{X_1 \lor X_2 \lor \ldots \lor X_N \lor Y_1 \lor Y_2 \lor \ldots \lor Y_M}$$

Resolution

$$\frac{X_1 \vee X_2 \vee \ldots \vee X_N \vee \beta, \quad Y_1 \vee Y_2 \vee \ldots \vee Y_M \vee \neg\beta}{X_1 \vee X_2 \vee \ldots \vee X_N \vee Y_1 \vee Y_2 \vee \ldots \vee Y_M}$$

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Resolution

Bob is a cat or lazy

Bob is a bird or he is not lazy

Bob is a cat or a bird

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$$A \Rightarrow B \equiv \neg A \vee B$$

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Conjunctive Normal Form

- Conjunction of **clauses** (disjunctions of literals)

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$$\alpha \Leftrightarrow \beta \equiv (\alpha \Rightarrow \beta) \wedge (\beta \Leftarrow \alpha)$$

$$\alpha \Rightarrow \beta \equiv \neg\alpha \vee \beta$$

$$\neg(\neg\alpha) \equiv \alpha$$

$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$$

$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$$

$$(\alpha \wedge (\beta \vee \gamma)) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$$

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Horn clauses

- Restricted logic. Sentences must be disjunctions with at most one positive literal
  - equivalent to conjunction implying an atom

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forward-chaining

backward-chaining

Summary

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- Inference in propositional logic: table, inference rules, resolution
- Horn clauses, forward/backward chaining