CS4804 Homework 4

Homework must be submitted electronically following the instructions on the course homepage. Make sure to explain you reasoning or show your derivations. You will lose points for unjustified answers, even if they are correct.

Written Problems

1. (Based on R+N 7.18) Consider the following sentence:

$$[(\textsc{Food} \Rightarrow \textsc{Party}) \vee (\textsc{Drinks} \Rightarrow \textsc{Party})] \Rightarrow [(\textsc{Food} \wedge \textsc{Drinks}) \Rightarrow \textsc{Party}]$$

   (a) Determine using enumeration whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.

   (b) Convert the left-hand and right-hand sides of the main implication into conjunctive normal form, showing each step.

2. (Based on R+N 8.9) This exercise uses the function MapColor and predicates In(x, y), Borders(x, y), and Country(x), whose arguments are geographical regions, along with constant symbols for various regions. In each of the following, we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence.

   (a) Paris and Marseilles are both in France.

      i. In(Paris $\wedge$ Marseilles, France)
      ii. In(Paris, France) $\wedge$ In(Marseilles, France)
      iii. In(Paris, France) $\vee$ In(Marseilles, France)

   (b) There is a country that borders both Iraq and Pakistan.

      i. $\exists$ c, Country(c) $\wedge$ Borders(c, Iraq) $\wedge$ Borders(c, Pakistan)
      ii. $\exists$ c, Country(c) $\Rightarrow$ [Borders(c, Iraq) $\wedge$ Borders(c, Pakistan)]
      iii. [$\exists$ c, Country(c)] $\Rightarrow$ [Borders(c, Iraq) $\wedge$ Borders(c, Pakistan)]
      iv. $\exists$ c, Borders(Country(c), Iraq $\wedge$ Pakistan)

   (c) All countries that border Ecuador are in South America.

      i. $\forall$ c, Country(c) $\wedge$ Borders(c, Ecuador) $\Rightarrow$ In(c, SouthAmerica)
      ii. $\forall$ c, Country(c) $\Rightarrow$ [Borders(c, Ecuador) $\Rightarrow$ In(c, SouthAmerica)]
      iii. $\forall$ c, [Country(c) $\Rightarrow$ Borders(c, Ecuador)] $\Rightarrow$ In(c, SouthAmerica)
      iv. $\forall$ c, Country(c) $\wedge$ Borders(c, Ecuador) $\wedge$ In(c, SouthAmerica)

   (d) No region in South America borders any region in Europe.

      i. $\neg$ [$\exists$ c, $\exists$ d, In(c, SouthAmerica) $\wedge$ In(d, Europe) $\wedge$ Borders(c, d)]
      ii. $\forall$ c, $\forall$ d, [In(c, SouthAmerica) $\wedge$ In(d, Europe)] $\Rightarrow$ $\neg$ Borders(c, d)]

iii. $\neg\ \forall$ c, $\textsc{In}$(c, SouthAmerica) $\Rightarrow \exists$ d, $\textsc{In}$(d, Europe) $\wedge \neg$ $\textsc{Borders}$(c, d)
iv. $\forall$ c, $\textsc{In}$(c, SouthAmerica) $\Rightarrow \forall$ d, $\textsc{In}$(d, Europe) $\Rightarrow \neg$ $\textsc{Borders}$(c, d)

3. (Based on R+N 8.23) For each of the following sentences in English, decide if the accompanying first-order logic sentence is a good translation. If not, explain why not and correct it. (Some sentences may have more than one error.)

   (a) No two people have the same social security number.

   $$\neg\exists x, y, n \quad \textsc{Person}(x) \wedge \textsc{Person}(y) \Rightarrow [\textsc{HasSS\#}(x, n) \wedge \textsc{HasSS\#}(y, n)]$$

   (b) John's social security number is the same as Mary's.

   $$\exists n \quad \textsc{HasSS\#}(\text{John}, n) \wedge \textsc{HasSS\#}(\text{Mary}, n)$$

   (c) Everyone's social security number has nine digits.

   $$\forall x, n \quad \textsc{Person}(x) \Rightarrow [\textsc{HasSS\#}(x, n) \wedge \textsc{Digits}(n, 9)]$$

   (d) Rewrite each of the above (uncorrected) sentences using a function symbol SS# instead of the predicate $\textsc{HasSS\#}$. E.g., if our knowledge base contains the information $\textsc{HasSS\#}(\text{John}, \texttt{123-456-7890})$, then $\text{SS\#}(\text{John}) = \texttt{123-456-7890}$.

4. (Based on R+N 9.9) Suppose you are given the following axioms:

   1. $0 \leq 3$.
   2. $7 \leq 9$.
   3. $\forall x \quad x \leq x$.
   4. $\forall x \quad x \leq x + 0$.
   5. $\forall x \quad x + 0 \leq x$.
   6. $\forall x, y \quad x + y \leq y + x$.
   7. $\forall w, x, y, z \quad w \leq y \wedge x \leq z \Rightarrow w + x \leq y + z$.
   8. $\forall x, y, z \quad x \leq y \wedge y \leq z \Rightarrow x \leq z$.

   Give a forward-chaining proof of the sentence $7 \leq 3 + 9$. Show only the steps that lead to success. Note that these axioms are written using our usual infix notation for arithmetic operators, but the relation $\leq$ and the operator $+$ can also be viewed as first-order logic concepts. Start by identifying what they are. Use only the axioms given here, not anything else you know about arithmetic from your experience as a human.