Homework problem set "Numerical math differs from math"

Problem I, 10 points.

Modify `numderivative.cc` to calculate the derivative $exp(x)'$ at $x = 1$ to within 0.05 % relative error by using: $f'(x) \approx (f(x + h/2) - f(x - h/2))/h$. What step size h will you need? What is the advantage of the above formula compared to the one we used in class [*i.e.* $f'(x) \approx (f(x+h) - f(x))/h$]? Using `numderivative.cc` , find the optimum h, and compare it to the optimum for the original `numderivative.cc` .

Problem II, 5 points.

One can come up with more and more clever approximations for $f'(x)$: the difference between the approximate $f'(x)$ and the mathematically exact derivative can be made arbitrary small for a given small value of h. In principle, there is no limit to how accurate your approximation can be in this respect. Now you use this approximate formula (which always contains subtraction of functional values) to numerically estimate $f'(x)$ on your computer. Is there a limit to the accuracy of the result? Why?

Problem III

In `numderivative.cc` replace the "exp(x)" with "sin(1/x)" where appropriate to obtain a numerical estimate for the derivative of $f(x) = sin(1/x)$ at

a. 5 points.) $x = 1/\pi$. Choose "h" so that the result is accurate to within at least 4 decimal points. What is your calculated result?

b. 5 points. What happens when you try the same code for $x = 10^{-20}/\pi$? Why? Use chain rule (and a change of variables) to re-formulate the problem into a mathematically equivalent one that is free from the defect. Modify the code and re-compute the derivative numerically. Show the results.