Homework Assignment 2, 45 points. Each student submits single PDF file, 3 pages max, typed. Font no smaller than 11pt. Make sure to label the axes of all your plots!

1 Problem set-up

This is a follow up on the "bug population" model discussed in class. As discussed in class, there are several ways to introduce non-trivial, oscillatory behavior into your model. An alternative to what we did would be to consider a second-order ODE, which, as you know, can always be reduced to two coupled differential equations. Biologically, this amounts to introducing an additional realism into the basic model: indeed, in real life, bugs are never left alone – birds eat them. In fact, this is how the bug population is kept in check. When birds are present, bug population p never becomes large enough for the quadratic term $\sim p^2$ to matter, so you can drop it from the model, keeping only $dp/dt = \alpha p$, $\alpha > 0$. Here t is the time variable. Instead, to describe the balance of the bug population, you add a term like βpx, where x is the bird population. The end result for the bug balance equation is $dp/dt = \alpha p + \beta px$. The meaning of the second term is that the change in the bug population is proportional to the number of bugs and the number of birds that eat the poor critters. To close the system of equations, you need to balance the birds. Assume that without food (bugs), any initial bird population simply dies off exponentially. This behavior of the bird population is described by the first term $A = A(x)$ in the bird balance $dx/dt = A + B$. To arrive at the form of B, it is reasonable to assume that the more bugs we have, the higher is the bird birth rate. So, $B = \delta px$.

2 Questions

Give a clear answer and provide an explanation. No explanation = no points.

Q1, 5 points What is the sign of β?

Q2, 5 points What is the sign of δ?

Q3, 5 points What is the mathematical form of $A(x)$? Comment on the sign of the prefactor (call it γ for consistency).

Q4, 5 points Write down the full system of 2 coupled ODEs that describes the bugs-birds balance and time-evolution of the populations. The system will have 4 parameters: $\alpha, \beta, \gamma, \delta$.

Q5, 5 points Find stationary solutions. What do they mean?

Q6, 10 points Use Mathematica to solve (numerically) the system of equations. Show the key lines of your code. Discuss at least two "sanity checks" for your numerical solution, show the plots.

Q7, 10 points Show that, for some values of parameters, you can have oscillations in the system. Show the plots. Can you give a hand-waving explanation for why?