# FP Foundations, Scheme (2) 

## In Text: Chapter 15

## Functional programming

- LISP: John McCarthy 1958 MIT
- List Processing => Symbolic Manipulation
- First functional programming language
- Every version after the first has imperative features, but we will discuss the functional subset


## LISP Data Types

- There are only two types of data objects in the original LISP
- Atoms: symbols, numbers, strings, ...
- E.g., a, 100, "foo"
- Lists: specified by delimiting elements with parentheses
- Simple lists: elements are only atoms
- E.g., (A B CD)
- Nested lists: elements can be lists
- E.g., ( $A(B C) D(E(F G)))$


## LISP Data Types

- Internally, lists are stored as singlelinked list structures
- Each node has two pointers: one to element, the other to next node in the list
- Single atom:
atom
- List of atoms: (abc)



## LISP Data Types

- List containing list (a (bc)d)



## Scheme

- Scheme is a dialect of LISP, emerged from MIT in 1975
- Characteristics
- simple syntax and semantics
- small size
- exclusive use of static scoping
- treating functions as first-class entities
- As first-class entities, Scheme functions can be the values of expressions, elements of lists, assigned to variables, and passed as parameters


## Interpreter

- Most Scheme implementations employ an interpreter that runs a "read-evalprint" loop
- The interpreter repeatedly reads an expression from a standard input, evaluates the expression, and prints the resulting value


## Primitive Numeric Functions

- Primitive functions for the basic arithmetic operations: $+,-, *, /$
-     + and * can have zero or more parameters. If * is given no parameter, it returns 1; if + is given no parameter, it returns 0

| Expression | Value |
| :---: | :---: |
| 42 | 42 |
| $(* 36)$ | 18 |
| $(+123)$ | 6 |
| $0($ sqrt 16) | 4 |

-     - and / can have two or more parameters
- Prefix notation


## Numeric Predicate Functions

- Predicate functions return Boolean values (\#T or \#F): =, <>, $\rangle,<,>=,<=$, EVEN?, ODD?, ZERO?

| Expression | Value |
| :---: | :---: |
| $(=1616)$ | \#T |
| $($ even? 29) | \#F |
| $(>10(* 24))$ |  |
| (zero? $(-10(* 25)))$ |  |

## Type Checking

- Dynamic type checking
- Type predicate functions
(boolean? x) ; Is $x$ a Boolean?
(char? $x$ )
(string? $x$ )
(symbol? $x$ )
(number? $x$ )
(pair? $x$ )
(list? $x$ )


## Lambda Expression

- E.g., lambda $(x)$ (* $\times x$ ) is a nameless function that returns the square of its given numeric parameter
- Such functions can be applied in the same ways as named functions
- E.g., $((\operatorname{lambda}(x)(* \times x)) 7)=49$
- It allows us to pass function definitions as parameters


## "define"

- To bind a name to the value of a variable: (define symbol expression)
- E.g., (define pi 3.14159)
- E.g., (define two_pi (* 2 pi))
- To bind a function name to an expression: (define (function_name parameters) (expression)
)
-E.g., (define (square $x$ ) (* $x \times$ ))


## "define"

- To bind a function name to a lambda expression
(define function_name (lambda_expression)
)
-E.g., (define square (lambda ( $x$ ) (* $\times x$ )))


## Control Flow

- Simple conditional expressions can be written using if:
-E.g. (if (<2 3) 4 5) $=>4$
-E.g., (if \#f 2 3) $=>3$


## Control Flow (cont'd)

- It is modeled based on the evaluation control used in mathematical functions: (COND
(predicate_1 expression)
(predicate_2 expression)
(predicate_n expression)
[ELSE expression]
)


## An Example

$$
f(x)=\left\{\begin{array}{l}
1 \text { if } x=0 \\
x^{*} f(x-1) \text { if } x>0
\end{array}\right.
$$

( define ( factorial $x$ )
( cond

$$
\begin{aligned}
& ((<\times 0) \# f) \\
& ((=x 0) 1)
\end{aligned}
$$

$$
\left(\# \dagger\left({ }^{\star} \times(\text { factorial }(-\times 1))\right)\right) \text {; or else }(\ldots)
$$

)

## Bindings \& Scopes

- Names can be bound to values by introducing a nested scope
- let takes two or more arguments:
- The first argument is a list of pairs
- In each pair, the first element is the name, while the second is the value/expression
- Remaining arguments are evaluated in order
- The value of the construct as a whole is the value of the final argument
- E.g. (let ((a 3)) a)


## let Examples

- E.g., (let ((a 3)
(b 4)
(square (lambda $(x)(* x x)$ ))
(plus +))
(sqrt (plus (square a) (square b))))
- The scope of the bindings produced by let is its second and following arguments


## let Examples

- E.g., (let ((a 3))
(let ((a 4)
(b a))
$(+a b)))=$ ?
- $b$ takes the value of the outer $a$, because the defined names are visible "all at once" at the end of the declaration list


## let* Example

- let* makes sure that names become available "one at a time"
- E.g., (let* $((x 1)(y(+x 1)))$

$$
(+x y)) \Rightarrow ?
$$

## Functions

- quote: identity function
- When the function is given a parameter, it simply returns the parameter
-E.g., (quote A) $=>$ A

$$
(\text { quote }(A B C)) \Rightarrow(A B C)
$$

- The common abbreviation of quote is apostrophe (')

$$
\begin{aligned}
-E . g ., & (A)=>A \\
& \left(\begin{array}{l}
(A B C)) \Rightarrow(A B C)
\end{array}\right.
\end{aligned}
$$

## List Functions

- car: returns the first element of a given list
- E.g., $\left(\operatorname{car}^{\prime}(A B C)\right) \Rightarrow A$
$\left(\operatorname{car}{ }^{\prime}((A B) C D)\right)=>(A B)$
( $\left.\operatorname{car}{ }^{\prime} A\right)$ ) ${ }^{\prime}$ ?
$\left(\operatorname{car}^{\prime}(A)\right)=>$ ?
(car $\left.{ }^{\prime}()\right)=>$ ?


## List Functions

- cdr: returns the remainder of a given list after the first element has been removed
- E.g., (cdr ' $(A B C))=>(B C)$
$\left(c d r^{\prime}((A B) C D)\right)=>(C D)$
$\left(c d r r^{\prime} A\right)=>$ ?
$\left(c d r^{\prime}(A)\right)=>$ ?
$\left(c d r{ }^{\prime}()\right)=>$ ?


## List Functions

- cons: concatenates an element with a list
- cons builds a list from its two arguments
- The first can be either an atom or a list
- The second is usually a list
- E.g., (cons ' $\left.\mathrm{A}^{\prime}()\right)=>(A)$
(cons ' $A$ ' $(B C)) \Rightarrow(A B C)$
(cons '( $\left.)^{\prime}(A B)\right)=>$ ?
(cons ' $\left.(A B)^{\prime}(C D)\right) \Rightarrow$ ?
- How to compose a list ( $A B C$ ) from $A, B$, and $C$ ?


## List Functions

- Note that cons can take two atoms as parameters, and return a dotted pair
-E.g., (cons 'A 'B) $=>$ (A . B)
- The dotted pair indicates that this cell contains two atoms, instead of an atom + a pointer or
a pointer + a pointer


## More Predicate Functions

- The following returns \#† if the symbolic atom is of the indicated type, and \#f otherwise
- E.g., (symbol? 'a) $\Rightarrow>$ \# $\dagger$
(symbol? '()) => \#f
- E.g., (number? '55) $=>$ \# $\dagger$
(number? 55) $=>$ \# $\dagger$
(number? '(a)) $\Rightarrow>\# f$
- E.g., (list? '(a)) $=>$ \# $\dagger$
- E.g., (null? '()) $=>$ \# $\dagger$


## More Predicate Functions

- eq? returns true if two objects are equal through pointer comparison
- Guaranteed to work on symbols
- E.g., (eq? 'A 'A) => \#T

$$
(e q \text { ? ' } A \text { ' }(A B)) \Rightarrow \# F
$$

- equal? recursively compares two objects to determine if they are equal
- The objects can be atoms or lists


## How do we implement equal?



```
(define (equal? lis1 lis2)
    (cond
        ((simple? lis1) (eq? lis1 lis2))
        ((simple? lis2) #F)
        ((equal? (car lis1) (car lis2))
                ((equal? (cdr lis1) (cdr lis2))
            (else #F)
    )
)
```


## More Examples

```
(define (member? atm lis)
    (cond
        ((null? lis) #F)
        ((eq? atm (car lis)) #T)
        (else (member? atm (cdr lis)))
        (define (append lis1 lis2)
    (cond
                ((null? lis1) lis2)
                (else (cons (car lis1)
                            (append(cdr lis1) lis2)))
)
```

What is returned for the following function?
(member? 'b '(a (b c)))

Is lis2 appended to lis1, or lis1 prepended to lis2?

An example: apply-to-all function
(define (mapcar fctn lis)
(cond
((null? lis) '())
(else (cons (fctn (car lis))
(mapcar fctn (cdr lis)) ))
)

## Project 3: A Scheme Parsing Program

- Consider the grammar $G=(S, N, T, P)$ where

$$
\begin{aligned}
& \mathrm{S}=(\text { Program }) \\
& \mathrm{N}=\text { ( statement list, statement, expr, symbol, op ) } \\
& \mathbf{T}=(\text { if, bool, then, while, id, const, }=,+,-, *, /) \\
& \mathrm{P}=(\text { Program } \quad \rightarrow \quad \text { statement_list } \\
& \text { statement_list } \rightarrow \text { statement statement_list } \\
& \rightarrow \quad \text { statement } \\
& \text { statement } \rightarrow \quad \text { if bool then statement_list } \\
& \rightarrow \quad \text { while bool statement_list } \\
& \rightarrow \quad \text { id }=\text { expr } \\
& \text { expr } \quad \rightarrow \quad \text { symbol op symbol } \\
& \text { symbol } \rightarrow \quad \rightarrow \quad \text { const } \\
& \text { op } \\
& \rightarrow \quad+|-|*| /
\end{aligned}
$$

- Write a Scheme program that correctly parses all valid programs in $L(G)$. The Scheme program will report (a) the total number of statements in the program, and (b) the maximum nested depth for a program.
- E.g., given ((id = id - const)), your program will output: (numberofstatements: 1 maximumdepth: 0)
- You can assume that
- You will be given only valid programs.
- Each program will be provided as a parenthesized list of statements, each of which is included in its own parentheses.
- The nesting of parentheses is used to indicate subordinate (or block) statement(s).


## Some Hints

- Define two functions to separately count (1) the number of statements and (2) the maximum nested depth
- When counting the number of statements, the function should
- Check whether the input parameter is an empty list
- If not, check whether the first element in the list is an empty list
- If not, obtain the first element in the list, check the statement type and proceed accordingly. The recursive function calls may be involved.
- When calculating the maximum nested depth, the function should do similar checks as what is mentioned above
- Notice that when an if-statement has both then- and else-branches, you need to compare the depth counts of both branches, obtains the larger number as the maximum nested depth at the current level


## On Rlogin ...

- You can use command "racket -i" to launch the interactive mode of Racket, and use "(exit)" to exit that mode
- Please name your main function "parser", which function takes only one argument.
- The program will be tested on Rlogin via "plt-r5rs < filename"
- The source file can be a .txt file

