Logic (or Declarative) Programming Foundations: Prolog

In Text: Chapter 12

Overview [1]

- Formal logic
- Logic programming
- Prolog

Logic Programming

- To express programs in a form of symbolic logic, and use a logic inferencing process to produce results
  - Symbolic logic is the study of symbolic abstractions that capture the formal features of logical inference
- Logic programs are declarative

Formal Logic

- A **proposition** is a logical statement or query about the state of the "universe"
  - It consists of objects and the relationship between objects
- **Formal logic** was developed to describe propositions, with the goal of allowing those formally stated propositions to be checked for validity

Symbolic Logic

- **Symbolic logic** can be used for three basic needs of formal logic
  - To express propositions,
  - To express the relationship between propositions, and
  - To describe how new propositions can be inferred from other propositions that are assumed to be true

Formal logic & mathematics

- Most of mathematics can be thought of in terms of logic
- The fundamental axioms of number and set theory are the initial set of propositions, which are assumed to be true
- **Theorems** are the additional propositions that can be inferred from the initial set

First-Order Predicate Calculus

- The particular form of symbolic logic that is used for logic programming is called **first-order predicate calculus**
- It contains **propositions** and **clausal form**

Propositions

- The objects in propositions are represented by simple terms
  - Simple terms can be either constants or variables
  - A **constant** is a symbol that represents an object
  - A **variable** is a symbol that can represent different objects at different times

Propositions

- The simplest propositions, which are called **atomic propositions**, consist of compound terms
- A **compound term** represents mathematical relation. It contains
  - a functor: the function symbol that names the relation, and
  - an ordered list of parameters

Compound Terms

- A compound term with a single parameter is a 1-tuple
  - E.g. man(jake)
- A compound term with two parameters is a 2-tuple
  - E.g., like(bob, steak)

Compound Terms

- All of the simple terms in the propositions, such as man, jake, like, bob, and steak, are constants
- They mean whatever we want them to mean
  - E.g., like(bob, steak) may mean
    - Bob likes steak, or
    - steak likes Bob, or
    - Bob is in some way similar to a steak, or
    - Does Bob like steak?
- Propositions can also contain variables, such as man(x)

Compound Propositions

- Two or more atomic propositions, which are connected by logical connectors

Name	Symbol	Example	Meaning
negation	$\neg$	$\neg a$	not a
conjunction	$\cap$	$a \cap b$	a and b
disjunction	$\cup$	$a \cup b$	a or b
equivalence	$\equiv$	$a \equiv b$	a is equivalent to b
implication	$\supset$	$a \supset b$	a implies b
	$\subset$	$a \subset b$	b implies a

Compound Propositions

- Quantifiers—used to bind variables in propositions
  - Universal quantifier: ∀
    ∀x.P — means "for all x, P is true"
  - Existential quantifier: ∃
    ∃x.P — means "there exists a value of x such that P is true"
  - Examples
    - ∀x.(manager(x) ⊃ employee(x))
    - ∃x.(mother(mary,x)) ∩ male (x))

Clausal Form

- **Clausal form** is a standard form of propositions
- It can be used to simplify computation by an automated system

Clausal Form

- A proposition in clausal form has the following general syntax:

$$\underbrace{B1 \cup B2 \cup \ldots \cup Bn}_{\text{consequent}} \subset \underbrace{A1 \cap A2 \cap \ldots \cap Am}_{\text{antecedent}}$$

- Consequent is the consequence of the truth of the antecedent
- Meaning
  - If all of the A's are true, then at least one B is true

Examples

- likes(bob, mcintosh) ⊂ likes(bob, apple) ∩ apple(mcintosh)
- father(john, alvin) ∪ father(john, alice) ⊂ father(alvin, bob) ∩ mother(alice, bob) ∩ grandfather(john, bob)

Predicate Calculus

- **Predicate calculus** describes collections of propositions
- **Resolution** is the process of inferring propositions from given propositions
- Resolution can detect any inconsistency in a given set of proposition

An Exemplar Resolution

- If we know:
  older(terry, jon) ⊂ mother(terry, jon)
  wiser(terry, jon) ⊂ older(terry, jon)
- We can infer the proposition:
  wiser(terry, jon) ⊂ mother(terry, jon)

Horn Clauses

- When propositions are used for resolution, only **Horn clauses** can be used
- A proposition with zero or one term in the consequent is called a **Horn clause**
  - If there is only one term in the consequence, the clause is called a **Headed Horn clause**
    - E.g., person(jake) ⊂ man(jake)
    - For stating **Inference Rules** in Prolog
  - If there is no term in the consequence, the clause is called a **Headless Horn clause**
    - E.g., man(jake)
    - For stating **Facts and Queries** in Prolog

Logic Programming Languages

- Logical programming languages are **declarative languages**
- **Declarative semantics**: It is simple to determine the meaning of each statement, and it does not depend on how the statement might be used to solve a problem
  - E.g., the meaning of a proposition can be concisely determined from the statement itself

Logic Programming Languages

- Logical Programming Languages are **nonprocedural**
- Instead of specifying *how* a result is computed, we *describe* the desired result and let the computer figure out how to compute it

An Example

- E.g., sort a list
  sort(new_list, old_list) $\subset$ permute(old_list, new_list) $\cap$ sorted(new_list)
  sorted(list) $\subset \forall$j such that $1 \leq j < n$, list(j) $\leq$ list(j+1)

where permute is a predicate that returns true if its second parameter is a permutation of the first one

Key Points about Logic Programming

- Nonprocedural programming sounds like the mere production of concise software requirements specifications
  - It is a fair assessment
- Unfortunately, logic programs that use only resolution face the problems of execution efficiency

Key Points about Logic Programming

- The best form of a logic language has not been determined
- Good methods of creating programs in logic programming languages have not yet been developed