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Complete Powerpoint Lecture Notes for
Computer Systems: A Programmer's Perspective (CS:APP)

Randal E. Bryant and David R. O'Hallaron

http://csapp.cs.cmu.edu/public/lectures.html

The book is used explicitly in CS 2505 and CS 3214 and as a reference in CS 2506.

Locality Example (1)

Claim: Being able to look at code and get a qualitative sense of its locality is a key skill for a professional programmer.

Question: Which of these functions has good locality?

```
int sumarrayrows(int a[M][N])
{
    int i, j, sum = 0;

    for (i = 0; i < M; i++)
        for (j = 0; j < N; j++)
            sum += a[i][j];
    return sum;
}
```

```
int sumarraycols(int a[M][N])
{
    int i, j, sum = 0;

    for (j = 0; j < N; j++)
        for (i = 0; i < M; i++)
            sum += a[i][j];
    return sum;
}
```


Layout of C Arrays in Memory

C arrays allocated in contiguous memory locations with addresses ascending with the array index:

```
int32_t A[10] = {0, 1, 2, 3, 4, ..., 8, 9};
```

7FFF99702320	0
7FFF99702324	1
7FFF99702328	2
7FFF9970232C	3
7FFF99702330	4
...	...
7FFF99702340	8
7FFF99702344	9

Two-dimensional Arrays in C

In C, a two-dimensional array is an array of arrays:

```
A[0]
A[1]
A[2]
```

```
int32_t A[3][5] = {
                 { 0,  1,  2,  3,  4},
                 {10, 11, 12, 13, 14},
                 {20, 21, 22, 23, 24}
      };
```

In fact, if we print the values as pointers, we see something like this:

```
A:       0x7fff22e41d30

A[0]:    0x7fff22e41d30
                          0x14   20₁₀
A[1]:    0x7fff22e41d44
                          0x14
A[2]:    0x7fff22e41d58
```

Layout of C Arrays in Memory

Two-dimensional C arrays allocated in *row-major order* - each row in contiguous memory locations:

```
int32_t A[3][5] =
 { { 0,  1,  2,  3,  4},
   {10, 11, 12, 13, 14},
   {20, 21, 22, 23, 24}
 };
```

7FFF22E41D30	0
7FFF22E41D34	1
7FFF22E41D38	2
7FFF22E41D3C	3
7FFF22E41D40	4
7FFF22E41D44	10
7FFF22E41D48	11
7FFF22E41D4C	12
7FFF22E41D50	13
7FFF22E41D54	14
7FFF22E41D58	20
7FFF22E41D5C	21
7FFF22E41D60	22
7FFF22E41D64	23
7FFF22E41D68	24

Layout of C Arrays in Memory

```
int32_t A[3][5] =
 { { 0,  1,  2,  3,  4},
   {10, 11, 12, 13, 14},
   {20, 21, 22, 23, 24},
 };
```

Stepping through columns in one row:

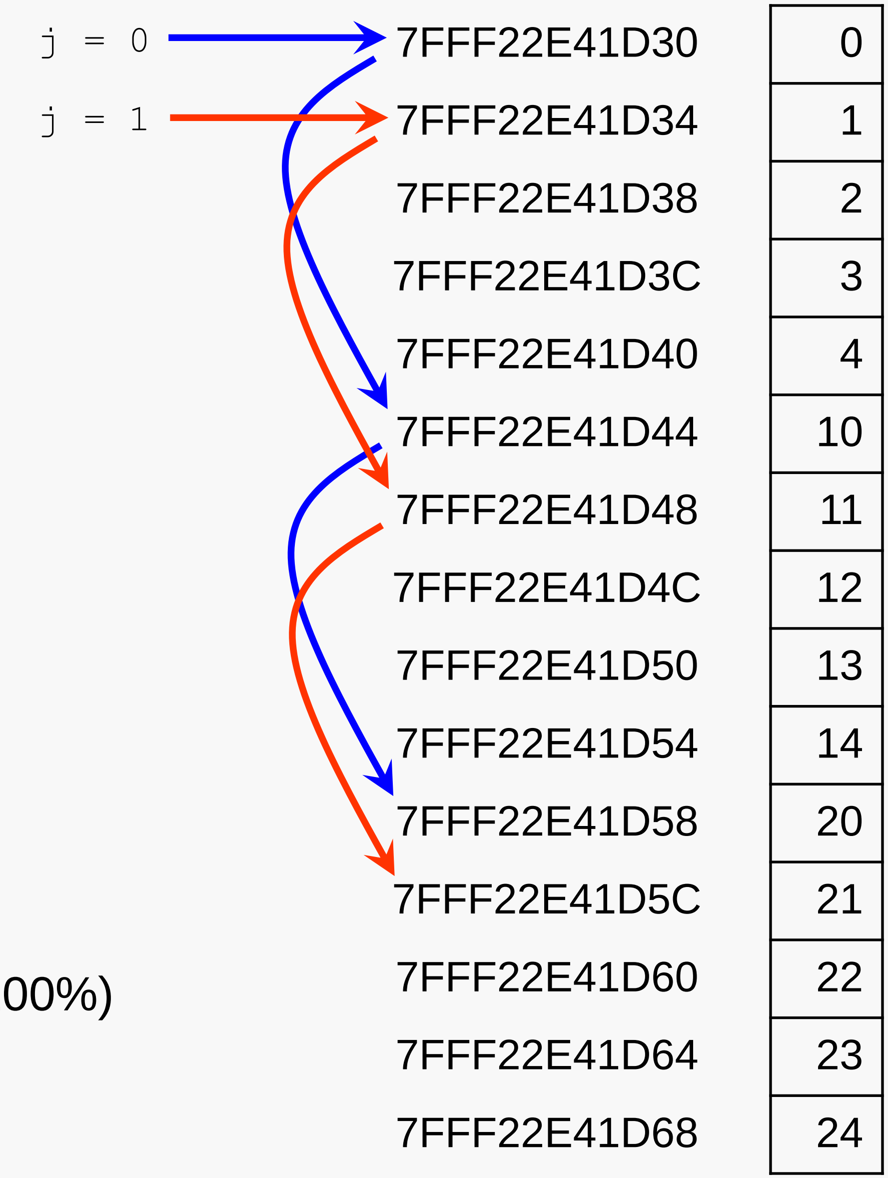
```
for (i = 0; i < 3; i++)
    for (j = 0; j < 5; j++)
        sum += A[i][j];
```

- accesses successive elements in memory
- if cache block size B > 4 bytes, exploit spatial locality compulsory miss rate = 4 bytes / B

	7FFF22E41D30	0
	7FFF22E41D34	1
i = 0	7FFF22E41D38	2
	7FFF22E41D3C	3
	7FFF22E41D40	4
	7FFF22E41D44	10
	7FFF22E41D48	11
i = 1	7FFF22E41D4C	12
	7FFF22E41D50	13
	7FFF22E41D54	14
	7FFF22E41D58	20
	7FFF22E41D5C	21
i = 2	7FFF22E41D60	22
	7FFF22E41D64	23
	7FFF22E41D68	24

Layout of C Arrays in Memory

```
int32_t A[3][5] =
 { { 0,  1,  2,  3,  4},
   {10, 11, 12, 13, 14},
   {20, 21, 22, 23, 24},
 };
```



Stepping through rows in one column:

```
for (j = 0; i < 5; i++)
   for (i = 0; i < 3; i++)
      sum += a[i][j];
```

accesses distant elements

no spatial locality!

compulsory miss rate = 1 (i.e. 100%)

Stride and Array Accesses

Stride 1

Stride 4

7FFF22E41D30	0
7FFF22E41D34	1
7FFF22E41D38	2
7FFF22E41D3C	3
7FFF22E41D40	4
7FFF22E41D44	10
7FFF22E41D48	11
7FFF22E41D4C	12
7FFF22E41D50	13
7FFF22E41D54	14
7FFF22E41D58	20
7FFF22E41D5C	21
7FFF22E41D60	22
7FFF22E41D64	23
7FFF22E41D68	24

Writing Cache Friendly Code

Repeated references to variables are good (temporal locality)

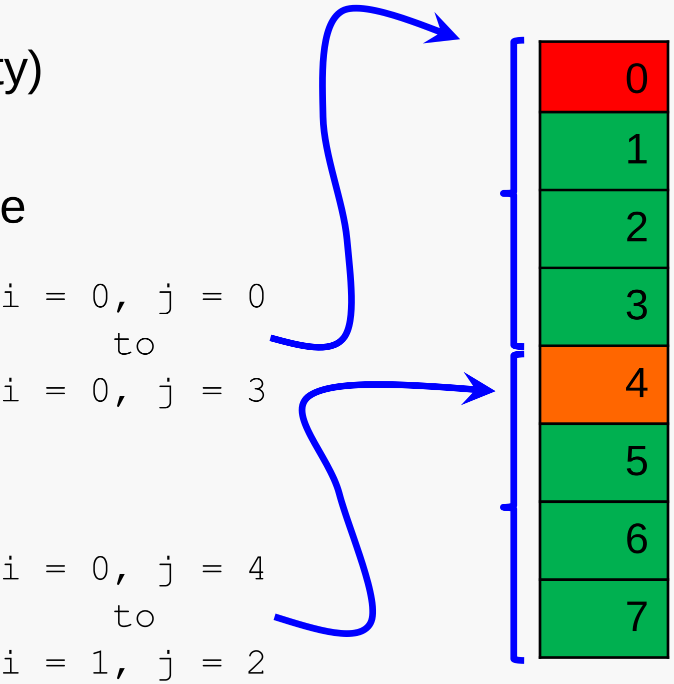
Stride-1 reference patterns are good (spatial locality)

Assume an initially-empty cache with 16-byte cache blocks.

```
int sumarrayrows(int a[M][N]) {

   int row, col, sum = 0;

   for (row = 0; row < M; row++)
      for (col = 0; col < N; col++)
         sum += a[row][col];
   return sum;
}
```



Miss rate = 1/4 = 25%

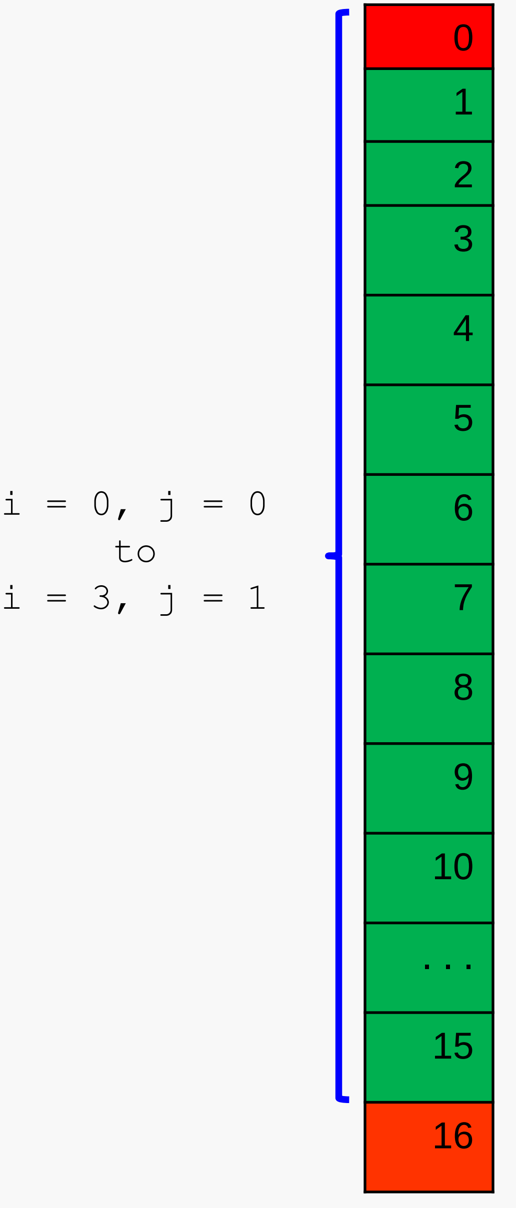
Writing Cache Friendly Code

Consider the previous slide, but assume that the cache uses a block size of 64 bytes instead of 16 bytes..

```
int sumarrayrows(int a[M][N]) {

   int row, col, sum = 0;

   for (row = 0; row < M; row++)
      for (col = 0; col < N; col++)
         sum += a[row][col];
   return sum;
}
```



Miss rate = 1/16 = 6.25%

Writing Cache Friendly Code

"Skipping" accesses down the rows of a column do not provide good locality:

```
int sumarraycols(int a[M][N]) {

   int row, col, sum = 0;

   for (col = 0; col < N; col++)
      for (row = 0; row < M; row++)
         sum += a[row][col];
   return sum;
}
```

Miss rate = 100%

(That's actually somewhat pessimistic... depending on cache geometry.)

Layout of C Arrays in Memory

It's easy to write an array traversal and see the addresses at which the array elements are stored:

```
int A[5] = {0, 1, 2, 3, 4};

for (i = 0; i < 5; i++)
   printf("%d:  %p\n",
            i,  &A[i]);
```

We see there that for a 1D array, the index varies in a stride-1 pattern.

```
i   address
-----------
0:    28ABE0
1:    28ABE4
2:    28ABE8
3:    28ABEC
4:    28ABF0
```

stride-1 : addresses differ by the size of an array cell (4 bytes, here)

Layout of C Arrays in Memory

```
int B[3][5] = { ... };
for (i = 0; i < 3; i++)
   for (j = 0; j < 5; j++)
      printf("%d %3d:  %p\n",
               i, j,   &B[i][j]);
```

We see that for a 2D array, the second index varies in a stride-1 pattern.

```
i-j order:

i   j     address
----------------
0   0:     28ABA4
0   1:     28ABA8
0   2:     28ABAC
0   3:     28ABB0
0   4:     28ABB4
1   0:     28ABB8
1   1:     28ABBC
1   2:     28ABC0
```

stride-1

But the first index does not vary in a stride-1 pattern.

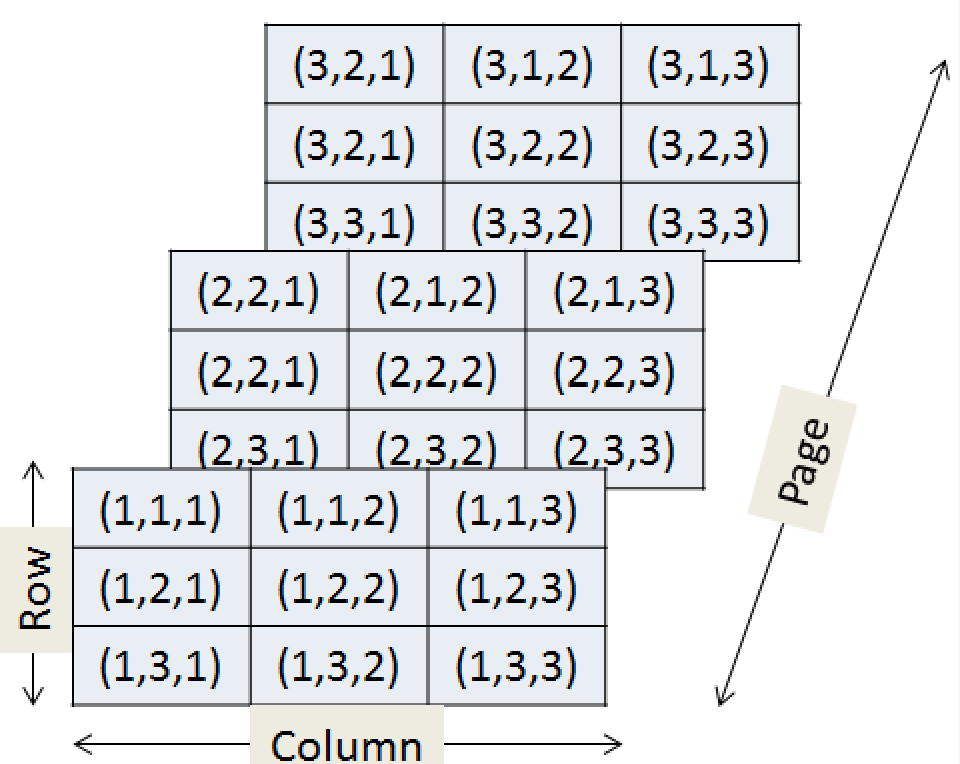
```
j-i order:

i   j     address
----------------
0   0:   28CC9C
1   0:   28CCB0
2   0:   28CCC4
0   1:   28CCA0
1   1:   28CCB4
2   1:   28CCC8
0   2:   28CCA4
1   2:   28CCB8
```

stride-5 (0x14/4)

3D Arrays in C

```
int32_t A[2][3][5] = {
    { {   0,    1,    2,    3,    4},
      {  10,   11,   12,   13,   14},
      {  20,   21,   22,   23,   24}},
    { {   0,    1,    2,    3,    4},
      { 110,  111,  112,  113,  114},
      { 220,  221,  222,  223,  224}}
  };
```



Locality Example (2)

Question: Can you permute the loops so that the function scans the 3D array `a[][][]` with a stride-1 reference pattern (and thus has good spatial locality)?

```
int sumarray3d(int a[N][N][N]) {

    int row, col, page, sum = 0;

    for (row = 0; row < N; row++)
        for (col = 0; col < N; col++)
            for (page = 0; page < N; page++)
                sum += a[page][row][col];
    return sum;
}
```

Layout of C Arrays in Memory

```
int C[2][3][5] = { ... };

for (i = 0; i < 2; i++)
   for (j = 0; j < 3; j++)
      for (k = 0; k < 5; k++)
         printf("%3d  %3d  %3d: %p\n",
                   i, j, k, &C[i][j][k]);
```

We see that for a 3D array, the third index varies in a stride-1 pattern:

But… if we change the order of access, we no longer have a stride-1 pattern:

```
i-j-k order:

i   j   k     address
-------------------
0   0   0:    28CC1C  } 0x4
0   0   1:    28CC20  } 0x4
0   0   2:    28CC24  } 0x4
0   0   3:    28CC28
0   0   4:    28CC2C
0   1   0:    28CC30
0   1   1:    28CC34
0   1   2:    28CC38
```

```
k-j-i order:

i   j   k     address
-------------------
0   0   0:    28CC24  } 0x3C
1   0   0:    28CC60  } 0x28
0   1   0:    28CC38  } 0x3C
1   1   0:    28CC74
0   2   0:    28CC4C
1   2   0:    28CC88
0   0   1:    28CC28
1   0   1:    28CC64
```

```
i-k-j order:

i   j   k     address
-------------------
0   0   0:    28CC24  } 0x14
0   1   0:    28CC38  } 0x14
0   2   0:    28CC4C  } 0x14
0   0   1:    28CC28
0   1   1:    28CC3C
0   2   1:    28CC50
0   0   2:    28CC2C
0   1   2:    28CC40
```

Locality Example (2)

Question: Can you permute the loops so that the function scans the 3D array `a[]` with a stride-1 reference pattern (and thus has good spatial locality)?

```
int sumarray3d(int a[N][N][N]) {

    int i, j, k, sum = 0;

    for (i = 0; i < N; i++)
        for (j = 0; j < N; j++)
            for (k = 0; k < N; k++)
                sum += a[k][i][j];
    return sum;
}
```

This code does not yield good locality at all.

The inner loop is varying the first index, worst case!

Writing Cache Friendly Code

Make the common case go fast

- Focus on the inner loops of the core functions

Minimize the misses in the inner loops

- Repeated references to variables are good (temporal locality)
- Stride-1 reference patterns are good (spatial locality)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories.

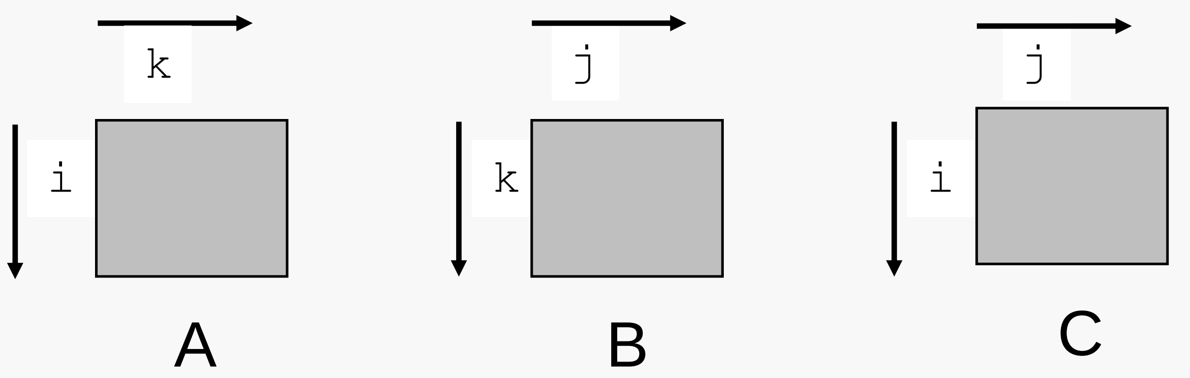
Miss Rate Analysis for Matrix Multiply

Assume:

- Line size = 32B (big enough for four 64-bit words)
- Matrix dimension (N) is very large
  - Approximate 1/N as 0.0
- Cache is not even big enough to hold multiple rows

Analysis Method:

- Look at access pattern of inner loop



Matrix Multiplication Example

Description:

- Multiply N x N matrices
- $O(N^3)$ total operations
- N reads per source element
- N values summed per destination

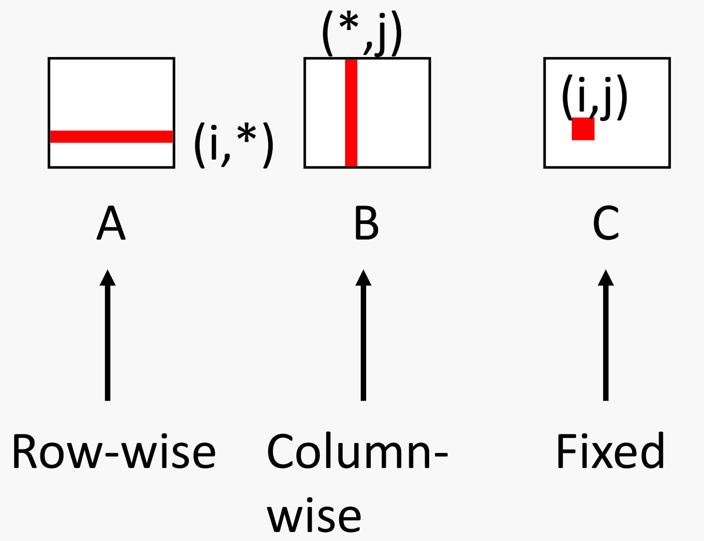
```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Variable `sum` held in register

Matrix Multiplication (ijk)

```
/* ijk */
for (i = 0; i < n; i++)  {
  for (j = 0; j < n; j++) {
    sum = 0.0;
    for (k = 0; k < n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:



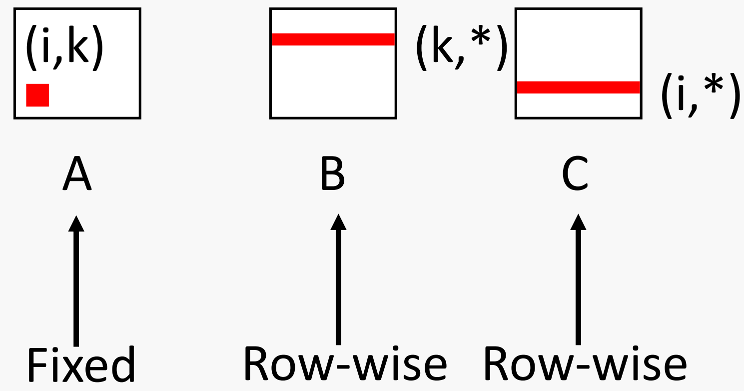
Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (kij)


```
/* kij */
for (k = 0; k < n; k++) {
  for (i = 0; i < n; i++) {
    r = a[i][k];
    for (j = 0; j < n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:



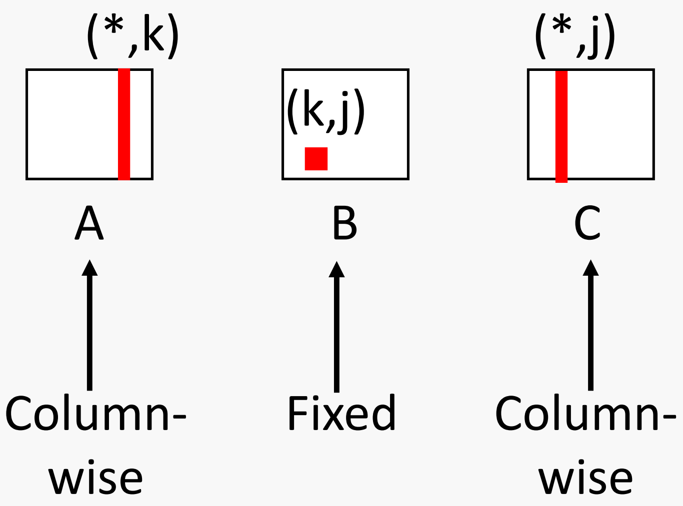
Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)

```
/* jki */
for (j = 0; j < n; j++) {
  for (k = 0; k < n; k++) {
    r = b[k][j];
    for (i = 0; i < n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Summary of Matrix Multiplication

```
for (i = 0; i < n; i++) {
   for (j = 0; j < n; j++) {
      sum = 0.0;
      for (k = 0; k < n; k++)
         sum += a[i][k] * b[k][j];
      c[i][j] = sum;
   }
}
```

ijk (& jik):
2 loads, 0 stores
misses/iter = A 0.25
B 1.00

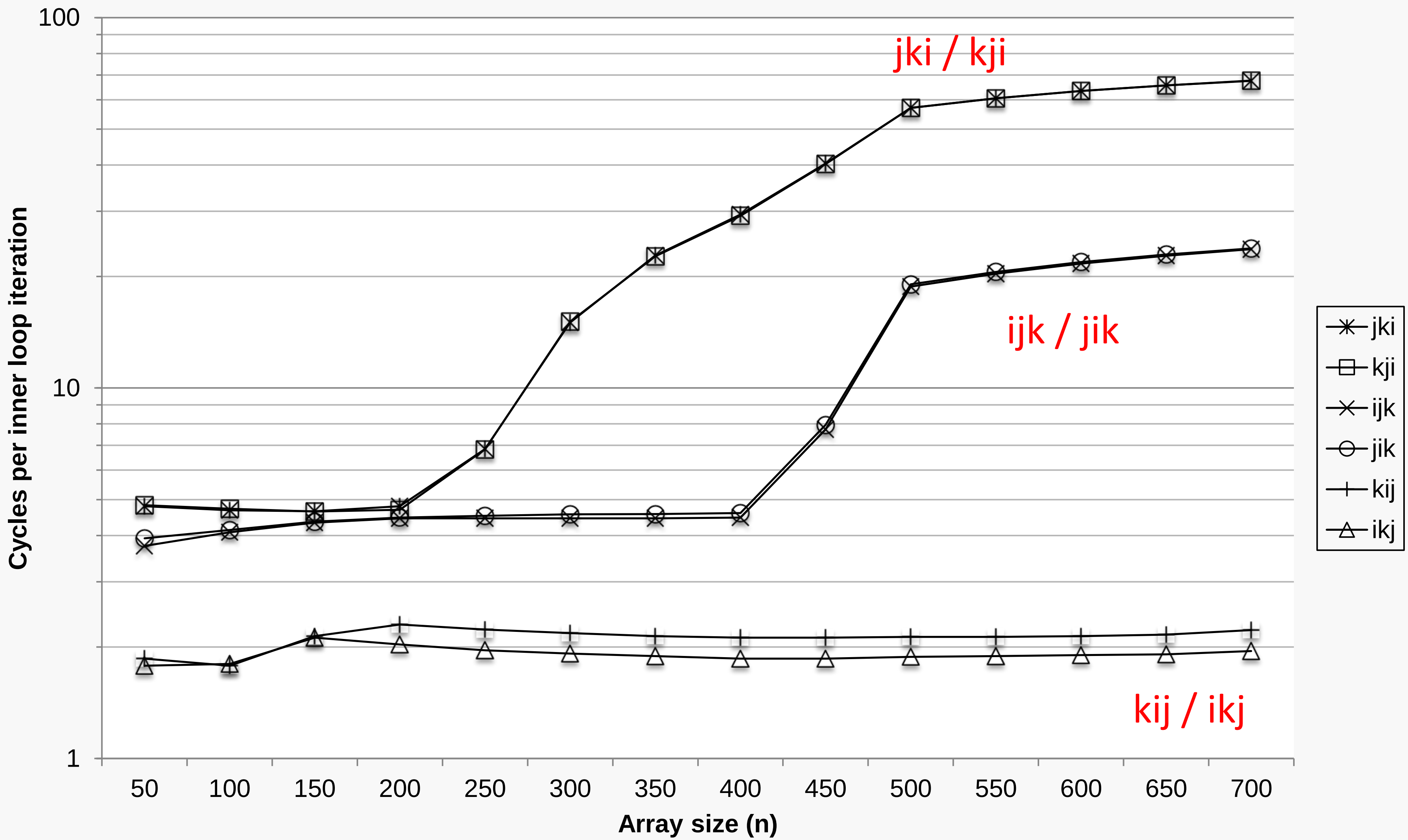
```
for (k = 0; k < n; k++) {
   for (i = 0; i < n; i++) {
      r = a[i][k];
      for (j = 0; j < n; j++)
         c[i][j] += r * b[k][j];
   }
}
```

kij (& ikj):
2 loads, 1 store
misses/iter = A 1.00
C 1.00

```
for (j = 0; j < n; j++) {
   for (k = 0; k < n; k++) {
      r = b[k][j];
      for (i = 0; i < n; i++)
         c[i][j] += a[i][k] * r;
   }
}
```

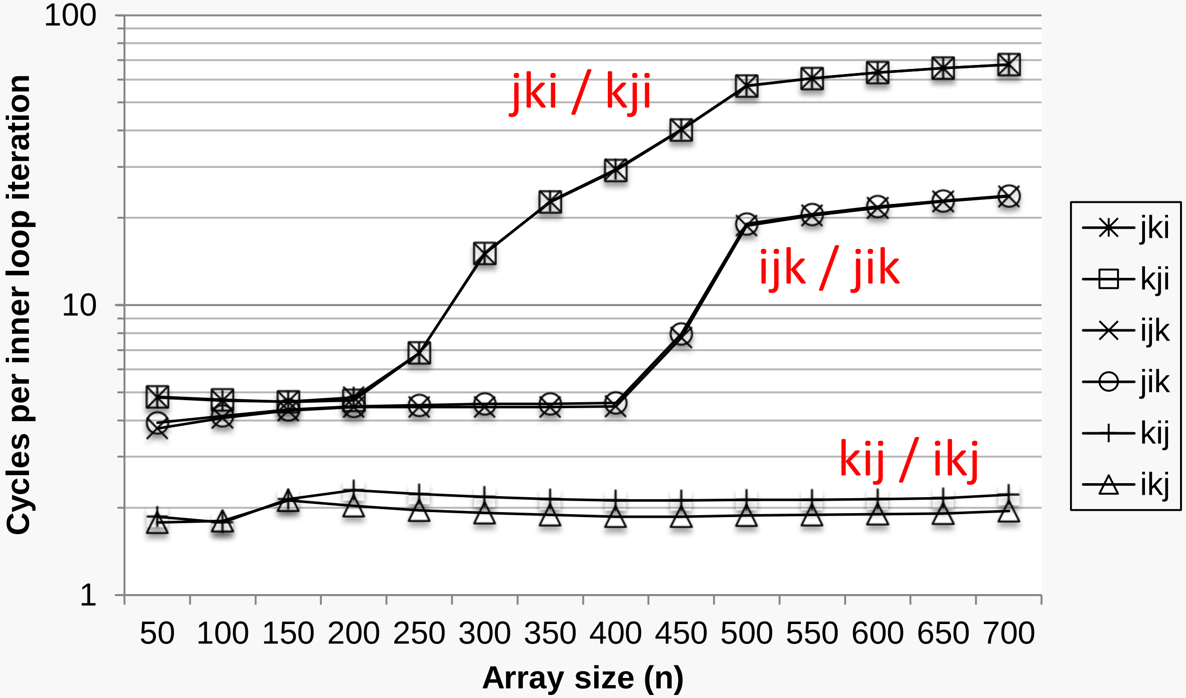
jki (& kji):
2 loads, 1 store
misses/iter = B 0.25
C 0.25

Core i7 Matrix Multiply Performance



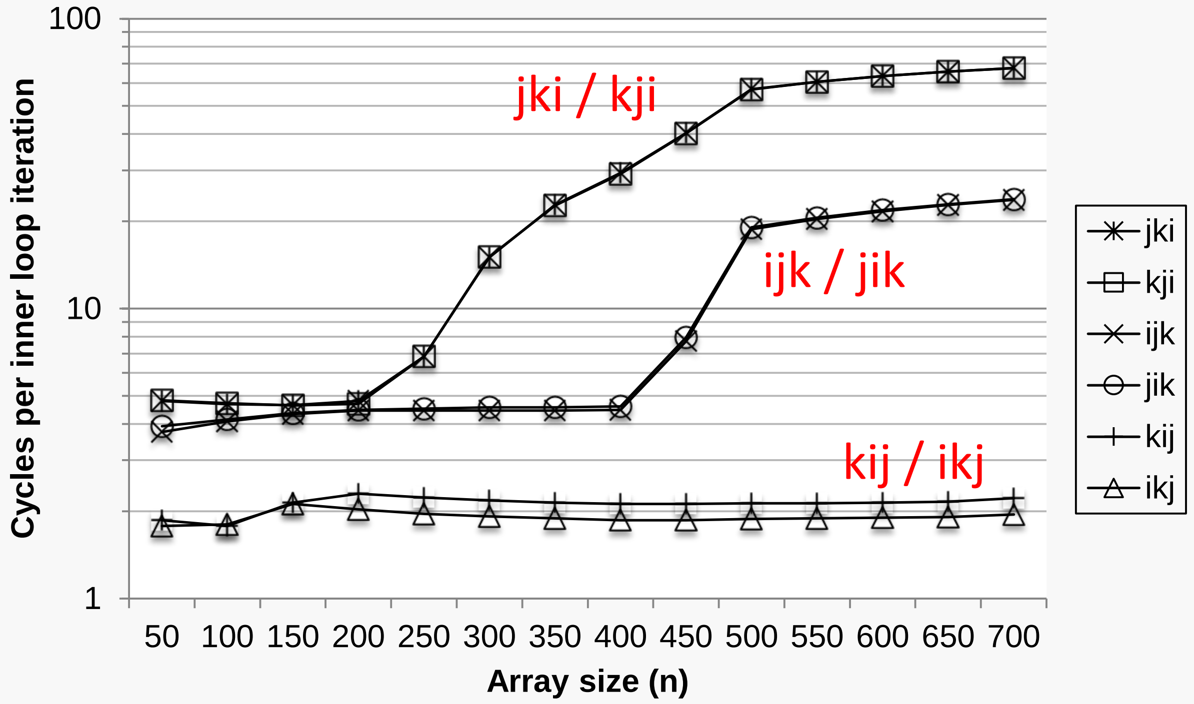
Be careful of the vertical scale here… it's actually rather messy.

Core i7 Matrix Multiply Performance



1. for large n, the kij/ikj versions run about 40 times faster than the jki/kji versions even though each version performs the same number of arithmetic operations!

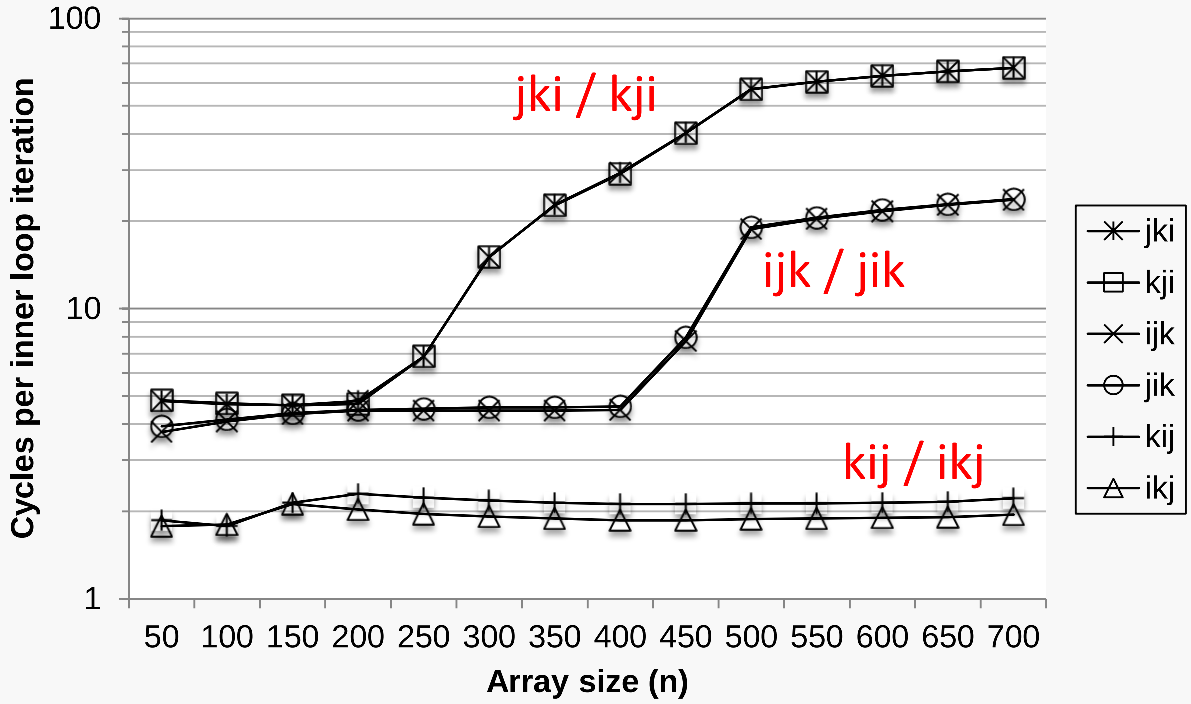
Core i7 Matrix Multiply Performance



2. pairs with the same miss count per iteration have essentially identical performance

   miss rate is a better predictor of performance than the number of memory accesses, at least in this example

Core i7 Matrix Multiply Performance



3. for the fastest pair, the cycle count is essentially constant as n increases

   the Intel cache prefetches intelligently, reacting to the stride-1 pattern quickly enough to keep up, even though the inner loop body is tight

Concluding Observations

- Programmer can optimize for cache performance
  - How data structures are organized
  - How data are accessed
    - Nested loop structure
    - Blocking is a general technique

- All systems favor “cache friendly code”
  - Getting absolute optimum performance is very platform specific
    - Cache sizes, line sizes, associativities, etc.
  - Can get most of the advantage with generic code
    - Keep working set reasonably small (temporal locality)
    - Use small strides (spatial locality)