Positional Notation

A *positional* or *place-value* notation is a numeral system in which each position is related to the next by a constant multiplier, called the *base* or *radix* of that numeral system.

The value of each digit position is the value of its digit, multiplied by a power of the base; the power is determined by the digit's position.

The value of a positional number is the total of the values of its positions.

So, in positional base-10 notation:

$$73901 = 7 \times 10^4 + 3 \times 10^3 + 9 \times 10^2 + 0 \times 10^1 + 1 \times 10^0$$

And, in positional base-2 notation:

$$10010000010101101 = 1 \times 2^{16} + 1 \times 2^{13} + 1 \times 2^7 + 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0$$

Why is the second example a cheat?

Vital Point

Do not confuse the representation with the number!

Each of the following examples is a representation of the same number:

255_{10}

11111111_{2}

FF_{16}

2010_{5}

377_{8}

3333_{4}

100110_{3}

Do not make the mistake of thinking that there is such a thing as "a base-10 number" or "a base-16 number".

There is a base-10 representation of every number and there is a base-16 representation of every number.

Converting from base-10 to base-2

Given a base-10 representation of an integer value, the base-2 representation can be calculated by successive divisions by 2:

73901	Remainder
36950	1
18475	0
9237	1
4618	1
2309	0
1154	1
577	0
288	1
144	0
72	0
36	0
18	0
9	0
4	1
2	0
1	0
0	1

10010000010101101_2

Converting from base-2 to base-10

Given a base-2 representation of an integer value, the base-10 representation can be calculated by simply expanding the positional representation:

$$10010000010101101_2 = 1\times 2^{16} + 1\times 2^{13} + 1\times 2^{7} + 1\times 2^{5} + 1\times 2^{3} + 1\times 2^{2} + 1\times 2^{0}$$

$$= 65536 + 8192 + 127 + 32 + 8 + 4 + 1$$

$$= 73901$$

Other Bases

Are analagous… given a base-10 representation of an integer value, the base-16 representation can be calculated by successive divisions by 16:

```
73901     Remainder
 4618        13 --> D
  288        10 --> A
   18         0
    1         2
    0         1
```

$120AD_{16}$

The choice of base determines the set of numerals that will be used.

base-16 (hexadecimal or simply hex)

numerals: 0 1 . . . 9 A B C D E F

Converting from base-2 to base-16

Given a base-2 representation of an integer value, the base-16 representation can be calculated by simply converting the nybbles:

```
1 0010 0000 1010 1101
1    2    0    A    D   : hex
```

The same basic "trick" works whenever the target base is a power of the source base:

```
10 010 000 010 101 101
 2   2   0  2    5   5  : octal
```

Important Bases in Computing

base-2	binary	0 1
base-8	octal	0 1 2 3 4 5 6 7
base-10	decimal	0 1 2 3 4 5 6 7 8 9
base-16	hex	0 1 2 3 4 5 6 7 8 9 A B C D E F