CS 2104 Homework Assignment 7. Group work. Each problem is worth 15 points

Problem 1. A bar of candy often comes organized into an m by n grid of rectangles so that it can be broken into smaller pieces. See the top of Figure 1, where the candy bar is a 3 by 4 grid of rectangles. In one move, you can pick up any piece of candy that has at least two rectangles and break it into two pieces along a horizontal or vertical line. See the two sample moves in Figure 1.

Prove that, no matter how you move, it will take the same number of moves to break the candy bar down to its individual rectangles. If the bar consists of $m \times n$ rectangles, what is that number of moves?

Problem 2. Let O be the origin of the plane. Imagine that five points, not equal to O, are placed on the plane. Prove that there are two of those points, P and Q, such that the angle $\angle POQ$ is acute.

View this problem as an application of the pigeonhole principle. What are the pigeonholes? What are the pigeons? Complete the argument.

Problem 3 You generate a random N-bit string, and compute $X = \sum_i^N x_i$, where x_i are the 0 and 1 entries of the string.

What is the probability that X is odd, if N is odd?

(15 points extra credit) What is the probability that X is odd if N is even? Hint: The straightforward solution of #1 will not work here, but the result from #1 may help. Induction may help too.

Submission   Strictly follow the "General Assignment Guidelines" (Group assignment) on the course web-site. Each partnership submits one document, with roles of each partner clearly indicated.
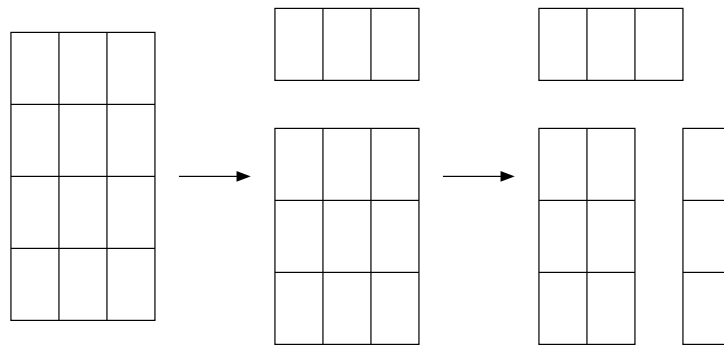
Figure 1: Sample moves for breaking a bar of candy.