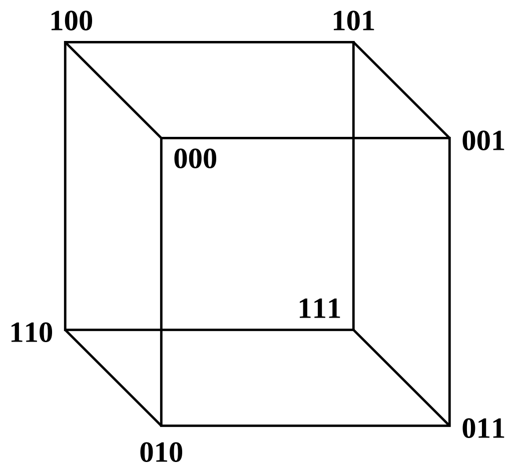
Random Walk on a Cube

CS 2104 Homework Assignment 4

Due: February 17, 2009

60 Points

The Problem. A random walk can be defined on a graph just as easily as we previously defined it on the integers. For this assignment, we will use the d-dimensional hypercube as the graph for the random walk. For $d \geq 1$, the *d-dimensional hypercube* is the graph with nodes that are bit strings (0's and 1's) of length d and an edge between u and v if u and v differ in exactly one bit. As a concrete example, choose $d = 3$. We get the *cube graph,* shown here:



For a node u of the cube, we can write $u = b_1b_2b_3$, a string of three bits. Its neighbors are $\overline{b_1}b_2b_3$, $b_1\overline{b_2}b_3$, $b_1b_2\overline{b_3}$, where $\overline{b_i}$ is the complementary bit to b_i. For example, if $u = 101$, then its neighbors are 001, 111, and 100.

For purposes of defining a random walk, the particle starts in some initial state S_0. If, after $i \geq 0$ steps, the particle is in state $S_i = b_1b_2b_3$, then in step $i+1$, it moves to state $\overline{b_1}b_2b_3$ with probability p; to state $b_1\overline{b_2}b_3$ with probability q; and to state $b_1b_2\overline{b_3}$ with probability r. It cannot stand still, so $p + q + r = 1$. Assume that $0 < p < 1$, $0 < q < 1$, and $0 < r < 1$. If $S_0 = 000$, then a possible sequence of states for the particle is

$$000, 001, 101, 001, 011, 010, 000, 100, 101, 100, 110, 111, 101, 111, 011.$$

For every node $b_1b_2b_3$, there is an opposite "corner" $\overline{b_1b_2b_3}$, its *antipode.* A random walk will eventually travel from any node to its antipode. The number of steps for a random walk to travel from S_0 to its antipode the first time is called the *antipode time* of the random walk. For example, the antipode time for the above random walk is the number of steps from the first 000 to the first 111, which is 11.

The Assignment. This assignment is to be done by the two assigned partners as a unit. The assignment is to write a program that will simulate a certain number of random walks on a cube for a given number of steps and that will estimate the average antipode time for the random walks. The program will be written in Java, C, or C++ as a single source file named `rwc.java` (Java), `rwc.c` (C), or `rwc.cpp` (C++).

The parameters for a simulation come from standard input as a single line of parameters, consisting of (1) the initial state S_0; (2) the value of p; (3) the value of q; (4) the value of r; and (5) the number of random walks to simulate. For example, the parameter line

```
001 0.25 0.35 0.40 2
```

specifies random walk simulations starting at $S_0 = 001$, with $p = 0.25$, $q = 0.35$, and $r = 0.40$, and repeated 2 times.

The output of the simulation goes to standard output. First, S_0 is printed. As the simulation proceeds, each new state is printed, one state per line. The first step that the antipode of S_0 is reached, the simulation of that random walk is terminated, a blank line is output, and a new simulation beginning at S_0 is started. This is repeated the specified number of times. Finally, the estimated average antipode time is printed (as below). Here is a sample output that uses the previous sample parameters.

```
001
101
111
011
010
110

001
000
100
000
010
011
111
110

Average antipode time is 6.0
```

Random Numbers. To complete the program, you will need a source of random numbers. See Homework Assignment 2 for the details. Suppose that the current state is $S_i = b_1 b_2 b_3$ and that you have a random number x uniformly distributed in $[0, 1]$, so $0 \le x \le 1$. If $0 \le x < p$, then the next state is $\overline{b_1} b_2 b_3$. If $p \le x < p + q$, then the next state is $b_1 \overline{b_2} b_3$. If $p + q \le x \le 1$, then the next state is $b_1 b_2 \overline{b_3}$.

Submission. The submission for this assignment must be the source file for your program. Each partnership uploads a single source file. The source file should be clearly commented and include the names of both partners. Your source file must be uploaded to Moodle by 11:00 PM on Tuesday, February 17.