

## Division Problems

$$\begin{array}{r}
 \text{xxx} \overline{) \text{xxxxxxxx}} \\
 \text{xxx} \phantom{0000000} \\
 \hline
 \text{xxxx} \phantom{00000} \\
 \text{xxx} \phantom{000000} \\
 \hline
 \text{xxxxx} \phantom{000} \\
 \text{xxxxx} \phantom{00} \\
 \hline
 \text{xxxxx} \phantom{0} \\
 \text{xxxxx} \\
 \hline
 \end{array}$$

↑  
DIV

Substitution:  $\text{xx}8\text{xx}$   
 Substitute a digit between 0-9 for each X. Initial digits are never 0.  
 Look for special features.

← PROD 1  
 ← PROD 2  
 ← PROD 3

## Division Problems

$$\begin{array}{r}
 \text{xxx} \overline{) \text{xxxxxxxx}} \\
 \text{xxx} \phantom{0000000} \\
 \hline
 \text{xxxx} \phantom{00000} \\
 \text{xxx} \phantom{000000} \\
 \hline
 \text{xxxxx} \phantom{000} \\
 \text{xxxxx} \phantom{00} \\
 \hline
 \text{xxxxx} \phantom{0} \\
 \text{xxxxx} \\
 \hline
 \end{array}$$

↑  
DIV

Special features:  
 PROD 1 not placed under the first 3 digits?  
 PROD 2 result of known multiplier: 8  
 Multiplying DIV by 8 yields a 3 digit number. Thus DIV must be a small number in the range 100-125 (since  $125 \times 8 = 1000$ ). So PROD 2 is a number between 800-992.

← PROD 1  
 ← PROD 2  
 ← PROD 3

## Division Problems

PROD 2

PROD 2

Special features:

PROD 2 is a number between 800-992.

It is subtracted from a 4 place number but yields a 2 place result. The only combination for which this can hold is when a 1 is carried to the second column to cancel a 9.

Therefore  $DIV * 8 > 900$  so  $DIV \geq 113 < 124$ .

XXXX

XXXX

-8xx-9xx

xx

xx

10xx

-9xx

xx

## Division Problems

$$\begin{array}{r}
 \text{1xx} \overline{) \text{xxxxxxxx}} \\
 \begin{array}{r}
 \uparrow \\
 \text{DIV}
 \end{array}
 \begin{array}{r}
 \text{xx8xx} \\
 \text{xxx} \leftarrow \text{PROD 1} \\
 \text{10xx} \\
 \text{9xx} \leftarrow \text{PROD 2} \\
 \text{xxxx} \\
 \text{xxxx} \leftarrow \text{PROD 3}
 \end{array}
 \end{array}$$

PROD 3 is the only product with 4 digits & when subtracted yields 0 so it must be identical to the number it is subtracted from. Which derived from PROD 2.

Multiplying DIV by final quotient digit yields a 4 digit number. Thus the final quotient digit  $> 8$  (which yielded a 3 digit number) therefore it is 9.

## Division Problems

Since  $113 \leq \text{DIV} < 124$ , PROD 3 must be:  
 $9 * 113 = 1017 \leq \text{PROD 3} < 1116 = 9 * 124$

$$\begin{array}{r}
 \text{xx8x9} \\
 \hline
 1\text{xx} \overline{) \text{xxxxxxxx}} \\
 \begin{array}{r}
 \uparrow \\
 \text{DIV}
 \end{array}
 \begin{array}{r}
 \text{xxx} \\
 \hline
 10\text{xx} \\
 \begin{array}{r}
 9\text{xx} \\
 \hline
 1\text{xxx} \\
 \begin{array}{r}
 1\text{xxx} \\
 \hline
 \end{array}
 \end{array}
 \end{array}
 \begin{array}{l}
 \leftarrow \text{PROD 1} \\
 \leftarrow \text{PROD 2} \\
 \leftarrow \text{PROD 3}
 \end{array}
 \end{array}$$

## Division Problems

PROD 2

Special features:

$$\begin{array}{r}
 10\text{xx} \\
 -9\text{xx} \\
 \hline
 1\text{x}
 \end{array}$$

PROD 2 is a number between 800-992.

Only 2 digits must be carried down.

What does the one beneath PROD2 imply?

The X's in column have a difference of one. And a one must be carried over to the 9 in column 3. The only pairs for which this holds are 0-9, 0-8, or 1-9 (in the last 2 cases a 1 must be carried from the first column). Thus PROD 2 is either 99X or 98X. Which implies DIV is either 123 or 124.

Note:

1. Whenever a number is carried down and the result is  $< \text{DIV}$  we place a 0 in the quotient digit.
2. The PROD 1 subtraction pattern is identical to the PROD 2 subtraction pattern. Thus the first quotient digit must be an 8.

## Division Problems

$$\begin{array}{r}
 \phantom{124} \overline{80809} \\
 124 \overline{)10020316} \\
 \underline{992} \phantom{00} \\
 1003 \phantom{00} \\
 \underline{992} \phantom{00} \\
 1116 \phantom{00} \\
 \underline{1116} \\
 0
 \end{array}$$

DIV is either 123 or 124 and the quotient must be 80809. By testing each of these possible divisors by the quotients to produce the products one will find that only 124 yields a result that satisfies all the constraints.