CS2104: Introduction to Problem Solving

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Goals of This Course

- Make you a better problem solver in general
  - Understand how you operate
  - Recognize limitations and pitfalls
  - Learn techniques that you can apply to solve problems
- Improve your ability to successfully complete the CS degree

Descriptive vs. Prescriptive

- Descriptive: How other people do it
- Prescriptive: Teach you how you should do it
  - Prescriptive is more useful to you than watching what others do.

What Motivated This Course?

We designed this course in hopes of:

- Improving students' ability to design
- Improving students' ability to develop algorithms
- Improving students' ability to plan (projects)
- Improving students' ability to test and debug
- Improving students' performance on tests
- Improving students' analytical abilities
- Improving students' ability to "argue" (proving)
- Improving students' ability with personal interactions

Guiding Philosphy

1. Problem solving is a skill (it can be learned). It is not an innate ability.
2. Problem solving is fundamentally about attitude and effort (the “problem-solving stance”).
3. The problem-solving stance isn’t something that you can just “turn on” when you need it for a test, etc. You have to live it – and successful people do just that.

Course Organization/Process

- Learn about yourself
- Learn problem-solving techniques
- Solve a wide variety of problems, so as to learn how to apply the techniques

What Kinds of Problems?

- Problems “in the large”: Engineering tasks
  - Lots of formal process, well developed
- Problems “in the small”: Puzzles, homework
  - Heuristics
- Success as a student
- Interpersonal problems
  - Take a “problem-solving” stance
- Analysis, construction, organization, process, understanding
- Communications

Know Yourself

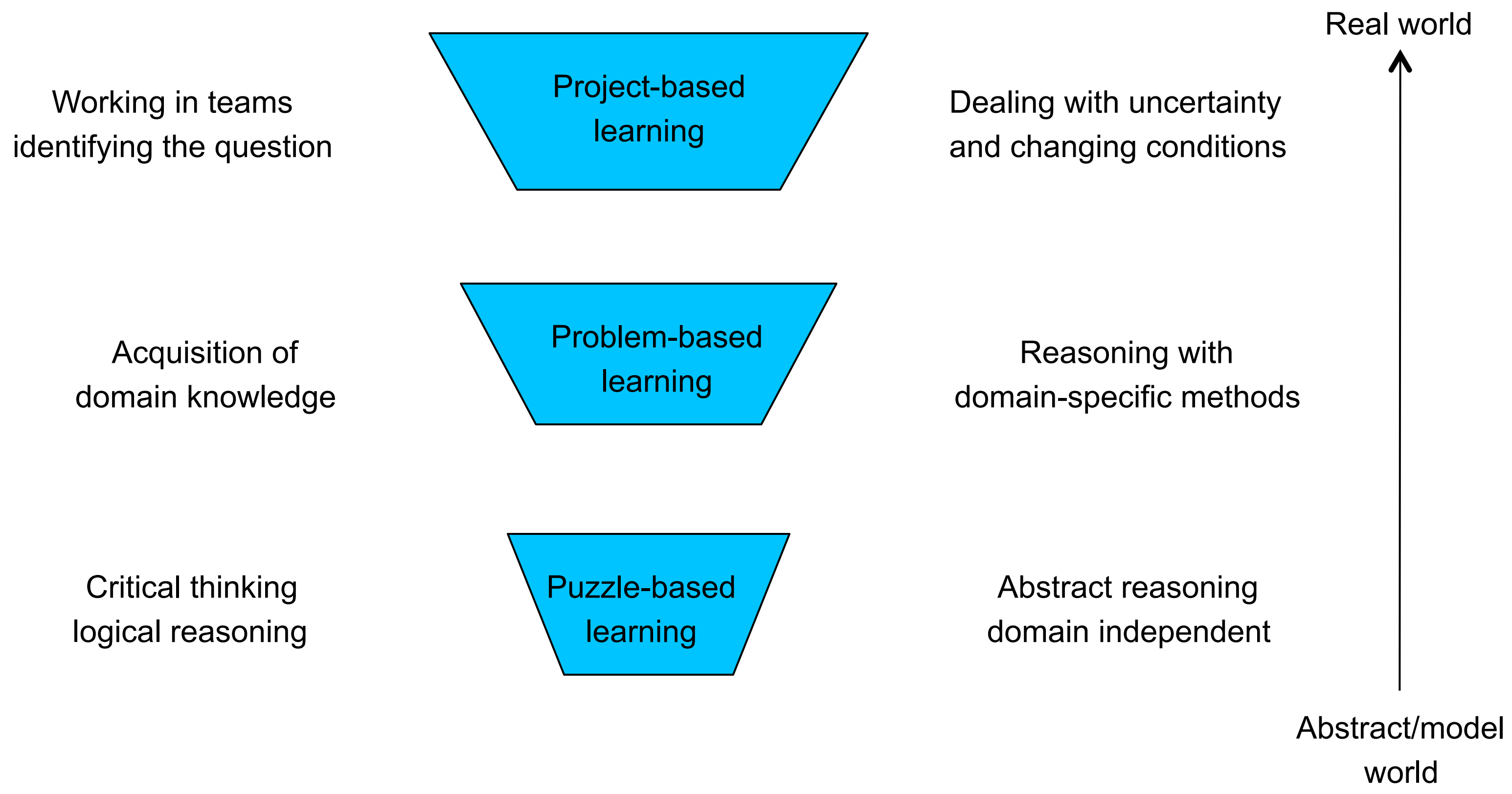
- Whimbey Analytical Skills Inventory (WASI)
- Myers-Briggs Personality Type
- Personality type tests

  http://www.humanmetrics.com/cgi-win/JTypes1.htm

  http://similarminds.com/jung.html

  (It is good to do a couple of different tests, results vary somewhat. Then, read the descriptions.)
- Soloman & Felder Index of Learning Styles

A Learning Hierarchy



N. Falkner, R. Sooriamurthi, and Z. Michalewicz, “Puzzle-based learning for engineering and computer science,” *IEEE Computer*, 43(4), 2010, pp. 20--28.

Errors in Reasoning

- Goal: Identify common types of errors and avoid them.
- Many of these come up in the WASI
  - A major reason for taking it is so that you can self-identify errors that you tend to make
- **Many points are lost on tests/homeworks in school come from errors in reasoning, not from lack of knowledge or skills.**
- You can train yourself to reduce making this sort of mistake.

Types of Errors

- Lack of knowledge or skill
  - This isn’t our focus, and its often not the issue
- Failure to observe and use all relevant facts of a problem
  - This might come from poor reading comprehension (which in turn often comes from rushing)
- Failure to approach the problem in a systematic manner, skip steps or jump to conclusion
- Failure to spell out relationships fully
- Being sloppy or inaccurate – making “simple” mistakes

Error Types Checklist

- Inaccuracy in Reading
- Inaccuracy in Thinking
- Weakness in Problem Analysis; Inactiveness
- Lack of Perseverance

Learning a Skill

- In general, to learn a skill (golf, driving car):
  1. Skill is demonstrated to student
  2. Student is directed and guided while practicing
- What about analytical reasoning skills?
  - It goes on inside the head – hard to demonstrate
  - Hard to direct and guide student

Thinking Aloud

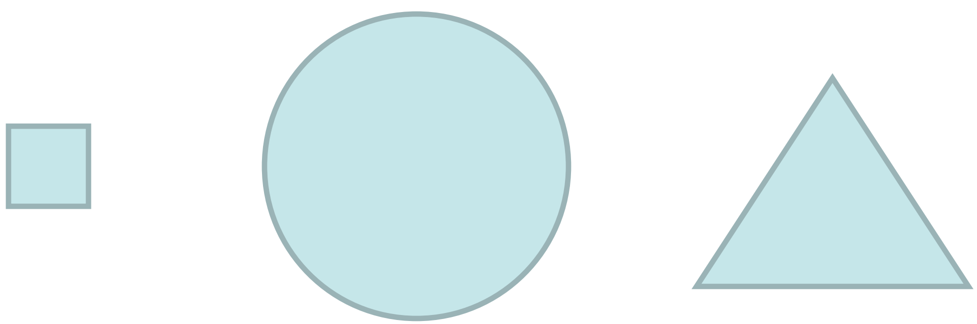
- The most effective way to expose the process is to verbalize our thinking process
  - This is hard work! Not our normal mode
  - Need to be careful to explain every step
- Demonstrate analytical reasoning by watching problem solvers solve problems while thinking aloud
- Practice problem solving by thinking aloud to a partner

Pairs Problem Solving

- We will use the technique of Whimbey & Lochhead
- The partners have distinct roles:
  1. One partner should read and <u>think aloud</u>.
     - On our homeworks, will be scribe as well
  2. The other partner is the listener
     - Continually check accuracy
     - Demand constant vocalization
- Thinking is a skill… but it is largely invisible
  – So we need to do everything possible to make it visible during this process

Problem 1

If the circle below is taller than the square and the triangle is shorter than the square, put a K in the circle. However, if this is not the case, put a T in the second tallest figure.



Characteristics of Good Problem Solvers

- Positive attitude
  - Belief that academic reasoning problems can be solved through persistence, as opposed to believing "either you know it or you don't"
  - Engage a confusing problem
- Concern for accuracy
  - Actively work to check your understanding
- Break the problem into parts
- Avoid guessing
  - And don't jump to conclusions
- Active in problem solving
  - Do more things as part of the process

Problem 2

If the word *sentence* contains less than 9 letters and more than 3 vowels, circle the first vowel. Otherwise circle the consonant that is farthest to the right in the word.

Role of the Listener

Crucial role, hard work. Not a passive role!

1. Continually check accuracy
   - Catch errors
   - Must work along/understand every step
   - Don’t let solver get ahead of him/herself
   - Point out errors, do not correct
2. Demand constant vocalization
   - Solver must spell out EVERY step
   - On homeworks, solver must make notes on EVERY step
3. Use Socratic questioning (Fogler/LeBlanc, p. 89—92)

Problem 3

Bill, Judy, and Sally have the occupations of teacher, plumber, and teamster but not necessarily in that order. Bill is shorter than Judy but taller than Sally. The plumber is the tallest and teamster is the shortest. What is Judy’s occupation?

Problem 4

If the second letter in the word *west* comes after the fourth letter in the alphabet, circle the letter A below. If it does not, circle the B.

A B

Problem 5

Give a rough estimate of the number of telephones on the Virginia Tech campus.

Problem 6

You drive a car at a constant speed of 40 mph from A to B, and on arrival at B, you return immediately to A but at a higher constant speed of 60 mph. What was your average speed for the whole trip?

Getting Started with a Problem

- "Eighty percent of success is showing up."
  - Woody Allen
  - "Success is 1% inspiration and 99% perspiration."
  - Thomas Edison
- To successfully solve any problem, the most important step is to get actively involved.
  - The Principle of Intimate Engagement: You must commit to the problem.
  - "Roll up your sleeves."
  - "Get your hands dirty."

Easy vs. Hard Problems

- Easy problems: See the answer
- Medium problems: See the answer once you engage
- Hard problems: You need strategies for coming up with a potential solution, sometimes for even getting started

Effective vs. Ineffective Problem Solvers

Effective: Believe that problems can be solved through the use of heuristics and careful persistent analysis

Ineffective: Believe ``You either know it or you don't.''

Effective: Active in the problem-solving process: draw figures, make sketches, ask questions of themselves and others.

Ineffective: Don't seem to understand the level of personal effort needed to solve the problem.

Effective: Take great care to understand all the facts and relationships accurately.

Ineffective: Make judgments without checking for accuracy

Mental Toughness

- Need the attributes of **confidence** and **concentration**
  - Confidence comes with practice
  - Attack a new problem with an optimistic attitude
- Unfortunately, it takes time
  - You can't turn it on and off at will
  - Need to develop a life-long habit

Engagers vs. Dismissers

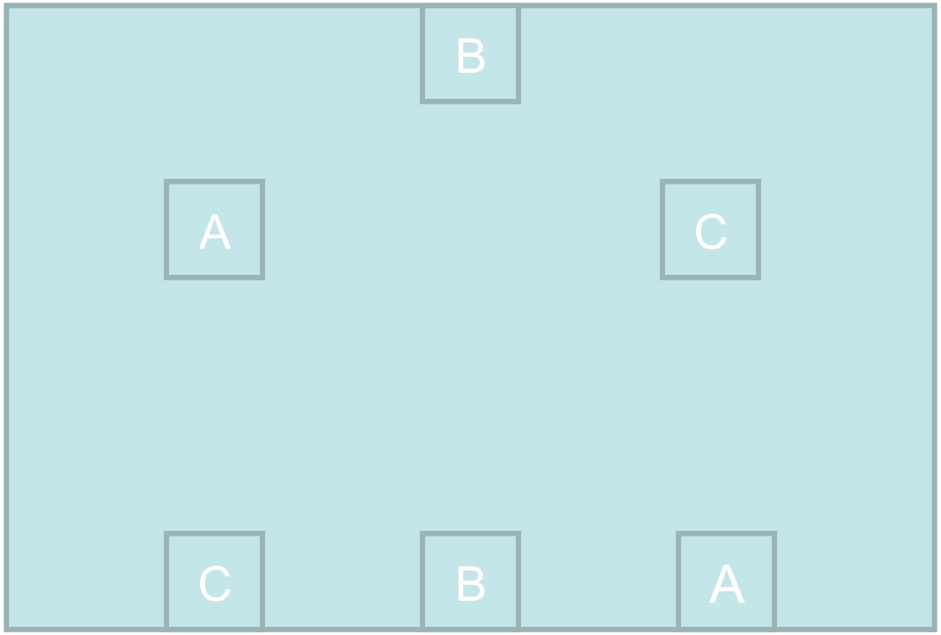
- Engagers typically have a history of success with problem solving.
- Dismissers have a history of failure.
- You might be an engager for one type of problem, and a dismisser for another.
- You can “intervene with yourself” to change your attitude of dismissal.

The Mental Block

- Many students do significant problem solving for recreation
  - Sodoku, computer games, recreational puzzles.
- These same students might dismiss math and analytical computer science problems due to a historical lack of success (the mental block)
- To be successful in life you will need to find ways to get over any mental blocks you have
- Learn to transfer successful problem-solving strategies from one part of your life to other parts.
  - Example: Writing is a lot like programming.

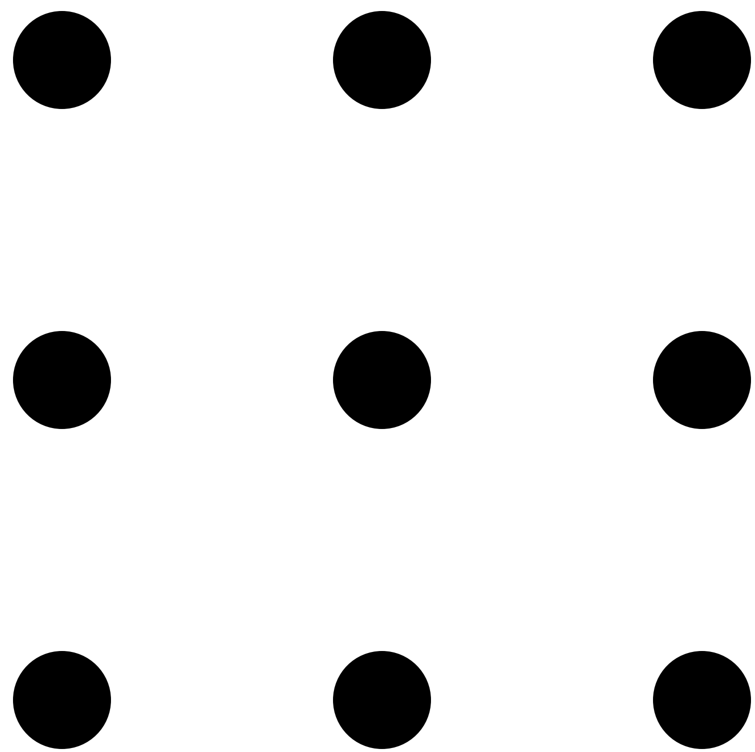
Example Problem

- Connect each box with its same-letter mate without letting the lines cross or leaving the large box.



Engagement Example

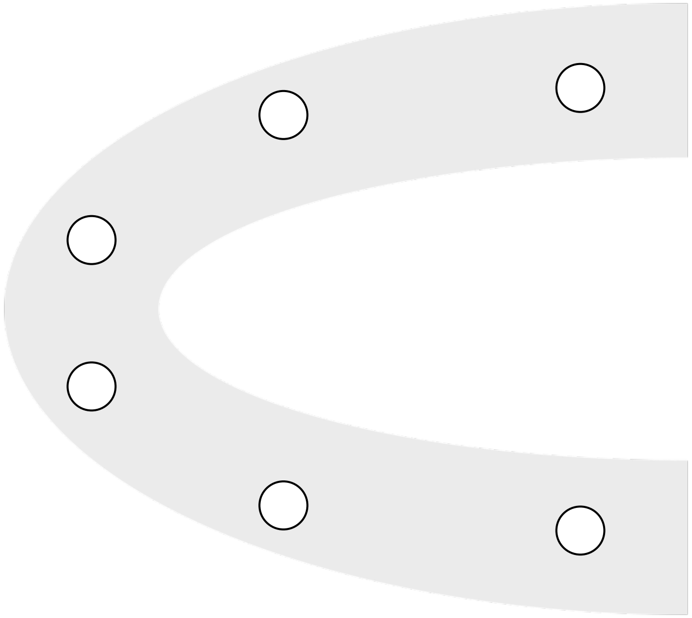
Draw a continuous polygonal line that passes through each circle exactly once – and use the fewest number of line segments you can manage:



Engagement Example

The picture below shows a horseshoe with six nail holes.

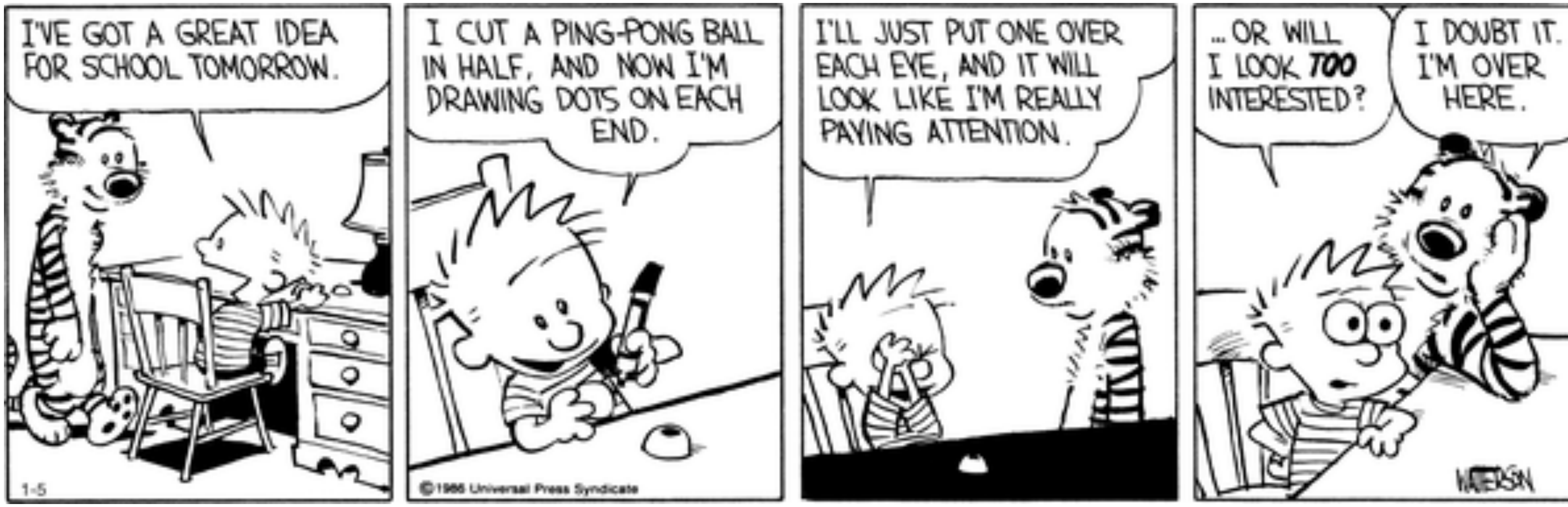
Can you make two straight cuts that result in six pieces, each containing a single nail hole?



Engagement Example

- Cryptoarithmetic problem

```
  A D
+ D I
---------
D I D
```


I'VE GOT A GREAT IDEA FOR SCHOOL TOMORROW.
I CUT A PING-PONG BALL IN HALF, AND NOW I'M DRAWING DOTS ON EACH END.
I'LL JUST PUT ONE OVER EACH EYE, AND IT WILL LOOK LIKE I'M REALLY PAYING ATTENTION.
... OR WILL I LOOK **TOO** INTERESTED?
I DOUBT IT. I'M OVER HERE.
WATTERSON
1-5
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MOM SAID I CAN'T GO OUTSIDE UNTIL I FINISH MY HOMEWORK. IF YOU'LL HELP ME, I'LL BE DONE FASTER. WHAT'S FIVE PLUS SEVEN?
I DON'T KNOW.
I DON'T EITHER.
THEN WRITE, "I DON'T KNOW."
HEY, THAT'S A TRUE ANSWER, ISN'T IT! I CAN WRITE THAT FOR **ALL** OF THESE! WE'RE DONE!
WE'D BETTER HAVE A LOOK AT OUR PRODIGY'S HOMEWORK.
WATTERSON
11-4

“Real World” Engagement Examples

- Repairing something (dryer, toaster, etc.)
- Dryer example: Clean it out
- Table example: Look for the loose parts
- Car seat example: Reattach spring wire
- “Taking the time”
- You **can** screw something up or do something dangerous. But often you are not faced with such a prospect.
  - Some domains require that you study/practice/build expertise to be effective
  - The act of engagement can help you build domain knowledge

Overcoming Procrastination

Writing/programming/project procrastination are all familiar behaviors.

Just sit down and write, don't care about the quality to start.

Write whatever part of the document/program appeals. Don't do it start to finish.

Do part of it at a time, over time.

- People don't write books, they write sections or pages.
- People don't write programs, they write functions, etc.

Set a schedule to work, including milestones, etc.

- Commit to someone outside if that helps.
- Invent deadlines if you are deadline-driven.
- Make the schedule a formal entity, and externalize.

Myers-Briggs Type Indicator

- Four dichotomies that define sixteen categories
  - Each is a continuum, not a binary choice
- This is not “what you are”
  - It is “right now, what you prefer” (and strength of preference)
  - For example, most introverts can operate in extrovert mode when needed.
- Results can vary from test to test or day to day by several points.
  - Results are generally consistent, between “adjacent” types
- Wikipedia has good articles for some types

Why Does it Matter?

- Presumably, different types are better/worse at different tasks
  - CS needs an unusually broad range of types to get everything done
  - Numerical analysis vs. HCI
  - Managers, architects, programmers, testers, documentation writers
- How do you best learn and work? Interact in organizations?
- Type/type interpersonal interactions
- Team building

What Type Am I?

- Depending on which test you take/ your current mood, you might end up assigned to different categories on different attempts.
- Testers often defer to the person on "best fit" category.
- Be careful when reading the descriptions
  - They tend to be general
  - They tend to be a bit flattering (which category type is for scatterbrained people? For couch potatoes?)
  - In general, readers tend to agree with any generic assignment that they are given (Forer effect)

Potential Failings

- Is it accurate?
- Unstable: Lots of variation in results between instruments and over time
- Does it make sense to say there are 16 personality types?
- Does it actually predict anything?

Four Dichotomies

- The words used for the poles on each of the four dichotomies have **technical** meanings
  - You can’t interpret what these mean using the everyday definitions of the words
  - A person isn’t “more judgmental” or “less perceptive” in these words’ everyday meaning

Introvert/Extravert [Attitude]

- Defines the source and direction of energy expression for a person.
  - Extravert has a source and direction of energy expression mainly in the external world. Act/reflect/act. Energy/motivation decline with inactivity.
  - Introvert has a source of energy mainly in the internal world. Reflect/act/reflect. Needs downtime after action to reflect.

Introvert/Extravert (Cont)

- These meanings are different from common use.
- You reflect before you act, and you might need plenty of “reflect time” after acting, but can still enjoy and interact at a party without being shy.
- An extravert might draw strength from acting rather than from reflecting, but that doesn’t tell us whether he is a “loud” person or not.

Sensing/iNtuition [Function]

- Defines the method of information perception
  - Sensing means that a person believes mainly information received directly from the external world – tangible and concrete facts drive patterns. More present oriented. Methodical, precise.
  - Intuition means that a person believes mainly information he or she receives from inside (books, memories) – how facts fit into the pattern. More future oriented. “Flash of insight.” Dislikes routine.
- Says what you prefer to focus on
  - Often need to use the opposite to “check”

Thinking/Feeling [Function]

- Defines how the person processes information (decision making). Both strive to make rational decisions. Both can be practiced/strengthened.
  - Thinking means that a person makes a decision mainly through logic/reason. More detached, impersonal.
  - Feeling means that, as a rule, he or she makes a decision holistically, including emotion. Look at from "inside" and strive to reach balance/harmony/ consensus with values. More personal, subjective.
  - "Heart vs. Head"
- You will trust your preferred approach better, but most have some ability to work in either mode.

Judging/Perceiving [Lifestyle]

- Defines how a person implements the information he or she has processed.
  - Judging means that a person organizes all his life events and acts strictly according to his plans. Prefers things decided. Prefers things on time. Might seem inflexible.
  - Perceiving means that he or she is inclined to improvise and seek alternatives. Likes to leave things open. More likely to push deadlines.

MB Example

- INTJ
  - Strength in each dimension (ex: mild I vs. E, mild N vs. S, moderate-strong T, strong J)
  - Occurrence in population (this one is 1-2%)
- One tending toward INTJ, on any given day/test, might register as ENTJ or ISTJ, but the person self-identifies with the INTJ descriptions.

Type Distribution In Population

ISTJ 11.6%	ISFJ 13.8%	INFJ 1.5%	INTJ 2.1%
ISTP 5.4%	ISFP 8.8%	INFP 4.4%	INTP 3.3%
ESTP 4.3%	ESFP 8.5%	ENFP 8.1%	ENTP 3.2%
ESTJ 8.7%	ESFJ 12.3%	ENFJ 2.4%	ENTJ 1.8%

What is the CS Personality?

- What is the “public perception” of CS?
- What is your perception?

	General	Engineering
E/I	70/30	33/67
N/S	30/70	47/53
J/P	50/50	61/39
F/T	50/50	26/74
F/T: Male	40/60	23/77
F/T: Female	60/40	39/61

Relevance to Education

- Different types prefer various teaching/testing styles
  - Sensing and Judging types prefer memorization and recall
  - iNtuition types prefer hypothesis/essay
  - Most in population are sensing
  - Most faculty are intuition
- Engineering students are split evenly N/S, but these groups have different needs

What Do Learning Styles Mean?

- Active learners need to do something with info – discuss, study in group
- Reflective learners need to think about it
- Sensors like facts, memorization, method
- Intuitors like innovation, can lose patience, need to avoid unnecessary mistakes
- Visual learners remember what they see
- Verbal learners remember what they hear/read
- Sequential learners work bottom up, know pieces but might not see relationships
- Global learners work top down, relate to the big picture, need a framework to fit the pieces to

Verbal Reasoning Problems

- For this type of problem, we need to parse the text into the proper steps
- Then we need to sort out the steps
- Since they can get long and complicated, we usually need to resort to a diagram (externalize the information)

VR Problem 1

Jose is heavier than Fred but lighter than Marty. Write their names in order of weight.

VR Problem 1 Solution

- To expose the reasoning process, we need to spell out the steps involved.
- Step 1: Jose is heavier than Fred… [He would be placed above Fred on the diagram.]
- Step 2: … but lighter than Marty. [So Marty is placed above Jose in the diagram.]

VR Problem 2

Jack is slower than Phil but faster than Val. Val is slower than Jack but faster than Pete. Write the names in order of speed.

VR Problem 2 Solution

- Step 1: Jack is slower than Phil… [He would be placed below Phil.]
- Step 2: … but faster than Val. [This says Jack is faster than Val. Val is added below Jack.
- Step 3: Val is slower than Jack… [We already knew this.]
- Step 4: But faster than Pete. [Val is faster than Pete, so Pete comes below Val.]

VR Problem 3

If Dumani and Fred are both richer than Tom, and Hal is poorer than Dumani but richer than Fred, which man is the poorest and which one is the next poorest? Write the names of all 4 men in order.

VR Problem 3 Solution

- Step 1: If Dumani and Fred are both richer than Tom…

  The problem does not indicate whether Dumani and Fred are actually equal to each other. So they can be represented at the same level for now, both above Tom.

- Step 2: … while Hal is poorer than Dumani but richer than Fred…

  This means that Dumani and Fred are not equal; Hal is between them with Dumani richest.

  Tom is poorest and Fred is next poorist.

VR Problem 4

Paul and Tom are the same age. Paul is older than Cynthia. Cynthia is younger than Hal. Is Paul older or younger than Hal – or can this not be determined from the information?

Other Diagrams

- Some problems are best supported by a 2D table.
- Some problems need another approach to organizing the information, such as a graph.

VR Problem 5

Three fathers – Pete, John, and Nick – have between them a total of 15 children of which 9 are boys. John has 1 more child than Pete, who has 4 children. Nick has 4 more boys than girls and the same number of girls as Pete has boys. How many boys each do Nick and Pete have?

VR Problem 5 Solution

	Boys	Girls	Total
Pete			
John			
Nick			
Total			

VR Problem 6

On a certain day I ate lunch at Tommy’s, took out 2 books from the library (The Sea Wolf and Martin Eden, both by Jack London), visited the museum and had a cavity filled. Tommy’s is closed on Wednesday, the library is closed on weekends, the museum is only open Monday, Wednesday, and Friday, and my dentist has office hours Tuesday, Friday, and Saturday. On which day of the week did I do all these things?

VR Problem 7

Boris, Irwin and Steven are engaged in the occupations of librarian, teacher, and electrician, although not necessarily in that order. The librarian is Steven's cousin. Irwin lives next door to the electrician. Boris, who knows more facts than the teacher, must drive 45 minutes to visit Irwin's house.

What is each man's occupation?

VR Problem 8

Sally loaned $7 to Betty. But Sally borrowed $15 from Estella and $32 from Joan. Moreover, Joan owes $3 to Estella and $7 to Betty. One day the women got together at Betty's house to straighten out their accounts. Which woman left with $18 more than she came with?

VR Problem 9

Lester has 12 times as many marbles as Kathy. John has half as many as Judy. Judy has half as many as Lester. Kathy has 6 marbles. How many marbles each do Lester and John have? You do not need to use algebra to solve this problem.

Six Myths about Reading

What speed-reading advocates say:

1. Don’t subvocalize when you read
2. Read only the key words
3. Don’t be a word-by-word reader
4. Read in thought groups
5. You can read at speeds of 1000 or more words a minute – without any loss of comprehension
6. Don’t regress or re-read

These things do not work.

There are no short cuts to comprehension!

Difficult material typically requires re-reading to understand.

Multitasking

See notes in the file Multitasking.pptx.

Analogies

- Analogy in communication promotes understanding.
- Use of analogy in nature supports creative problem solving (mechanical inventions from biological analogy)
  - I can solve this problem in a way similar to how that problem was solved.

Analogy and Problem Solving

Working analogy problems requires

- Spelling out ideas fully
- Formulating precise relationships of facts
- Developing correspondences between ideas
- Comparing relationships for similarities and differences

These skills are central to all problem solving.

Simple Analogy Example

Gills are related to *fish* as *lungs* are related to *humans*.

Restate:

- *Gills* are used for breathing by *fish*.
- *Lungs* are used for breathing by *humans*.
- (Where did “used for breathing” come from?)

Define a “relationship sentence”:

- ______ are used for breathing by _______.

Analogy 1

The key issue in analogy problems is picking the proper relationship sentence.

Carpenter is to *saw* as *plumber* is to *wrench*.

- A ____ is a ____.
- A ____ cuts wood with a ____.
- A ____ uses a tool called a ____.

Analogy 2

Stewardess is to *airplane* as *waitress* is to *restaurant*.

- A ____ is a(n) ____.
- A ____ works in a(n) ____.
- A ____ gives safety instructions in a(n) ____.

Analogy 3

Guitar is to *pick* as *fiddle* is to *bow*.

- A _____ is played with a _____.
- A _____ is plucked with a _____.
- A _____ is a _____.

Analogy 4

Fence is to *garden* as *bumper* is to *car*.

- A _____ helps protect a _____.
- A _____ keeps trespassers out of a _____.
- A _____ surrounds a _____.

Analogy 5

20 is related to 10 as 50 is related to 40.

- _____ is ten more than _____.
- _____ is twice _____.
- _____ is one-half of _____.

Analogy 6

50 is related to 48 as 67 is related to 64.

- _____ is two more than _____.
- _____ is larger than _____.
- _____ is smaller than _____.

Define the Relationships

- *Mouth* is to *talk* as *hand* is to *grasp*.
- 6 is related to 2 as 21 is related to 7.
- 70 is related to 30 as 35 is related to 15.
- *Arrive* is to *depart* as *find* is to *lose*.
- *Roots* are to *plant* as *mouth* is to *animal*.
- *Peacock* is to *bird* as *tuxedo* is to *suit*.
- 50 is related to 20 as 90 is related to 60.

Standard Test Analogy Problems

Now we look at the standard form of analogy problems on tests.

- One pair is given, you pick another pair that has the same relationship.
- It helps if you can define a relationship sentence.

Analogy 1

Thermometer is to *temperature* as _____ is to _____.

a) telescope : astronomy
b) clock : minutes
c) scale : weight
d) microscope: biologist

Analogy 2

Horse is to *animal* as _____ is to _____.

a) cow : milk
b) farm : pig
c) oak : wood
d) saddle : stallion

Analogy 3

2 is to 6 as _____ is to _____.

a) 6 : 2
b) 12 : 36
c) 3: 1
d) 12 : 60

Analogy 4

- *Pack* is to *wolves* as _____ is to _____.

a) alphabet : letters
b) wheel : spokes
c) garage : cars
d) aquarium : fish

Analogy 5

Same idea, just a different format.

_____ is to dollar as year is to _____.

a) money : calendar
b) penny : century
c) dime : month
d) savings : century

Analogy 6

Try each choice. If the relationships are different, the answer is wrong. If the relationships are unclear, then hold the answer to reconsider.

_____ is to *cave* as *car* is to _____.

a) Modern : primitive
b) Stone : steel
c) Primitive : modern
d) Apartment house : horse

Heuristics for Problem Solving (in the small)

- Heuristic: A rule of thumb, a way of doing things that might or might not work
- Goal of problem-solving heuristics: Help us to overcome our own limitations
  - Motivation
  - Working memory
  - Insight
  - Process
  - Emotions

The Mind

Three things that your mind does:

1. Receives/processes external information
2. “Displays” stored information
3. Manipulates information

It tends not to do more than one of these well at a time.

Limited “bandwidth” of attention

Externalizing

- After motivation and mental attitude, the most important limitation on your ability to solve problems is biological:
  - Working memory is 7 +/- 2 “pieces of information.”
- You can't change this biological fact. All you can do is take advantage of your environment to get around it.
- That means, you must put things into your environment to manipulate them.
- Externalize: write things down, manipulate aspects of the problem (correct representation).

Example

A rubber ball has the property that, on any bounce, it returns to one-third of the height from which it just fell. Suppose the ball is dropped from 108 ft. How far has the ball traveled the fourth time it hits the ground?

Externalizing

- In this example, drawing the picture left your mind free to concentrate on problem solving.
- Not drawing is probably hopeless, too much to keep track of.
- To be effective, the drawing needs to be set up right – a diagram of some sort makes a big difference.

Example

- Remember these numbers: 483 and 627
- Now, look away and multiply them in your head.

Example

A rectangular board is sawed into two pieces by a straight cut across its width. The larger piece is twice the length of the smaller piece. This smaller piece is cut again into two parts, one three times the length of the other. You now have three pieces of board. The smallest piece is a 7-inch square. What was the original area of the surface of the board?

Straight-line Problems

Problems along one dimension: distance, money, etc.

John has a pretty good salary. In fact if the salary of his older brother, Bob, were suddenly doubled, John would make only 100 dollars less than Bob. Bob's current salary is 50 dollars more than that of the youngest brother, Phil. John makes 600 dollars per week. What is Phil's salary?

Draw a line and put the information onto the line.

A Logic Problem

Tom, Dick, Harry, and Al are married to May, Jane, Sue, and Bea, though not necessarily in that order. Jane, who is Dick's sister, has five children. Tom and his wife want to wait a few more years before starting a family. Tom has never introduced his wife to Sue, who is carrying on an extramarital affair with Dick. (May is considering telling Dick's wife about it.) Dick and Harry, by the way, are twin brothers. Who is married to whom?

Matrix Problems

- How can we organize this information?
  - Matrix works well in this case
- Can work on one row/column (e.g., figure out who X is married to.
- Can work one fact at a time.
  - In this case, we will get pretty far. But we'll be left with a 2 by 2 box for Harry/Al and Jane/Sue. How do we break it?
  - We need to relate two facts to infer that Dick, Harry, Jane are all siblings.

Example

Three boys, Joey, Jimmy, and Pete, have between them nine quarters and a total of $2.55 in quarters and nickels. Joey has three nickels, and Jimmy has the same number of quarters. Jimmy has one coin more than Joey, who has four coins. How many nickels each do Jimmy and Pete have?

Hand-Shaking Problem

An anthropologist and her husband attended a party with four other married couples. Whenever two people shook hands, the woman recorded that each of the two people shook hands one time. In that way, for all of them (including herself and her husband), she obtained the total number of times that each person shook hands. She noted that one didn't shake hands with one's own spouse. Then she observed: If she didn't count herself, the other nine people all shook hands a different number of times. That is, one person didn't shake any hands, one shook only once, up to one shaking hands of all eight of the others.

Q: How many times did her husband shake hands?

Hand-Shaking Problem

- This one is difficult. Its tough to engage.
- But there are things that can be figured out. You need to play with it awhile.
- Hint: Can the anthropologist's husband be the one who shook hands 8 times?
- Bigger hint: Draw out a table!

Function-Generating Problems

A gambler bets 3 dollars on the first spin of a roulette wheel. Each time he loses he doubles his bet. He has lost n times in a row. How do we express A_{n+1}, the amount of his bet for the next (the n+1) spin?

- Perhaps you can do this in your head, but making a table will illustrate the process.

A Table

# of spins	Amount bet, A
1	3
2	$3 * 2 = 6$
3	$3 * 2^2 = 12$
4	$3 * 2^3 = 24$
5	$3 * 2^4 = 48$

Pattern: $A_{n+1} = 3 * 2^n$

Handball Tournament Problem

- In a single-elimination tournament with *n* participants, how many games must be played?
- Solve by building up a table of values in the series.

Induction Proofs

- Ideally, table generating can then get enough insight to make a good guess about the conclusion of a series.
- Later you will formalize this by using induction to prove that your guesses are correct.

(Aside: Why should CS students take a math minor? Not because they need the math itself. Rather, because it teaches you to think straight.)

Reading Comprehension

- This is critical to our success, both as a student and in later life.
- So it benefits us to do better at it.
- As a reader, visualizing the material is the most powerful way to “see” what is being communicated.

Example

A seashore is a better place than the street. At first it is better to run than to walk. You may have to try several times. It takes some skill but it’s easy to learn. Even young children can have fun.

Once successful, complications are minimal. Birds seldom get too close. Too many people doing the same thing, however, can cause problems. One needs lots of room. Beware of rain; it ruins everything. If there are no complications, it can be very peaceful. A rock will serve as an anchor. If things break loose from it, however, you will not get a second chance.

Context

- The passage probably doesn’t make sense until you know what it is about (flying kites). Then you can visualize it.
- If you were given a test on your comprehension of the passage, the result would depend greatly on whether you knew the context or not.

Visualization and Comprehension

Even when discussing numeric problems, “seeing” the relationship is important.

As Jack walked to town he met three beggars. He gave them each 4 dollars. That left only 2 dollars for himself, but he didn’t care. He was happy.

How much money did Jack start with?

Example

Jack stuffed the 16 dollars into his wallet and decided to go to town to buy a toy. He left his house and walked a half-mile when he met the beggar. The man seemed so poor that Jack gave him half the money in his wallet. About every half-mile he was approached by another beggar, each more wretched than the last. He met the third one just at the outskirts of town. Jack gave to each one half the money in his wallet. As he left the third begger and entered the town he saw that he had only 2 dollars left but he didn't care. He was happy.

Passage Comprehension

Eighty students served in this experiment on problem solving. Each student received one of four similar problems (referred to as problems A, B, C, and D). Since we were interested in the effects of distraction, half the students worked on their problem with music playing; half worked in silence. The ten students in each condition consisted of one eight-year-old, four ten-year-olds, and five twelve-year-old children.

Questions

1. How many conditions were there? What were they?
2. Why does the author refer to ten students?
3. How many ten-year-olds served in this experiment?

The questions are easy… but you might not have gotten the necessary information out of the passage from unguided reading. It is hard to train yourself to pull out all the information without being primed by a question to answer.

A table of information might help.

Another Passage

Thirty-six students (eighteen males and eighteen females) served in an experiment on problem solving. Each of these students received three problems, A, B, and C. Since each subject was receiving all three problems, the sequence of problem presentation was varied. All possible permutations (BCA, CAB, etc.) were used. Three males and three females were assigned to each of the six different sequences.

Questions

- Why were there six different sequences? Could there have been more than this number? What were these six sequences?
- Did the number of students used, thirty six, strike you as unusual? Why did the experiment use such a number instead of a nice, round number like thirty or forty? What other numbers might the experimenter have used?

Step Back…

Q: What is the most important problem that you have to solve over the next few years?

Problem Statement

- Each person will have a slightly different problem statement or emphasis
- The problem is to:
  - Get a degree
  - With the desired GPA
  - With the desired knowledge of the field
  - With the desired knowledge from other fields
  - With desired “other” experiences (academic or otherwise)
  - Without going insane.

How to Succeed as a Student

- Take a “problem solving stance” with this problem of succeeding as a student.
- What are problems that students run into?
- What are strategies for academic success?

Engage the Problem

- Being a student is a job.
  - With all of the professional responsibilities that holding a job implies
  - A full time student has a full time job (typically 40-60 hours/week is expected)
  - To succeed as a student requires applying the same sense of professionalism that you would apply to any other job.

Four Keys to Success

- Learn to network
- Learn to focus
- Learn to present
- Learn to play the game
  - Don’t change a winning game, but always change a losing one.

Learn to Network

- The #1 problem for many students is lack of interaction with faculty/staff
  - Not only can they help you academically, they also help you professionally
- #2 problem for many students is lack of interaction with other students in the discipline
  - Build a support environment/community

Learn to Focus: Organization

- Communications/email
- Deadlines in-the-large
  - Degree/semester deadlines
- Deadlines in-the-small
  - Daily/weekly planners
  - To-Do lists
  - Good habits trump good memory

Learn to Focus: Environment

- Doing hard work requires a conducive environment
  - Setting yourself up for success
- Certain types of work don't need full attention
  - Music, etc. might be OK
- Other types of work need complete focus
  - Writing (prose or code), most homework problems, hard debugging.

Persistence vs. Productivity

- Some students just don’t work hard enough
- Some students work hard… but don’t get good results. Possible causes:
  - Poor networking skills
  - Poor organizational skills
  - Poor working environment

Learn to Present

- The primary goal of communication is to invoke the desired response in your audience.
  - Email communications
  - Tests
  - Homework
  - Writing
  - Proof and argument
- We will discuss communication more later in the semester.

Learn to Play the Game

- Being a good student is a (learned) skill, not an (innate) ability
- Get the easy points
  - Never shortchange easy assignments or classes
  - A little investment (or reallocation) of time could raise your overall score
- Learn the testing game
- Learn time management/stress management
- If you need testing time accommodation, don't be shy about getting it.

Change a Losing Game

- Felder & Silverman Learning styles scale
  - What is your preferred approach to learning?
  - Be prepared to adjust to various styles of course
  - Seek adjustment in the course conditions if practical

Resources

- Faculty/staff!!
- Writing Center
- Counseling Center
  - Mental health
  - Eating disorders, substance abuse
  - Stress
  - Career counseling
  - Study skills, etc.

Thomas-Kilmann Conflict Mode Instrument

- The TKI indicates your general preferred approach to conflict resolution.
- There are two dimensions:
  - assertiveness (satisfy yourself),
  - cooperativeness (satisfy others).
- There are pros and cons to various approaches.
- When you understand how you tend to function, you can improve on it.

TKI Modes

- Five modes (dimension scores in parentheses):
  - Accommodating (1/9): Set aside your objectives to satisfy others.
  - Competing (9/1): Attempt to fulfil your objectives at expense of others.
  - Avoiding (1/1): Seek to avoid conflict altogether (withdraw).
  - Compromising (5/5): Seek balance in conflict.
  - Collaborating (9/9): Seek to go beyond conflict to help both sides.

What Scores Mean

- Differences in scores indicate strength of preference.
  - Highest score is your dominant preference.
  - Most people can use all five modes to some degree.
- Low differences mean ease of moving between modes.

Interpersonal Problem Solving

- Goal: When dealing with people, take a “problem-solving stance”.
- This will increase your chance of a satisfactory outcome.
- In contrast, our own emotions might make us blind to solutions, or unable to implement recognized solutions.

An Interpersonal Problem

John, a student living in the dorms, has for a neighbor a fellow who parties and plays music set at full volume almost every night into the small hours of the morning. John, a serious student, is unable to sleep for the noise. He clearly has a problem, one caused by another person.

Interpersonal Problems

- How does this differ from our earlier types of problems?
  - Another person’s (conflicting) goals/needs are involved
  - The solution does not depend solely on intellectual skill
  - Our own emotions tend to get in the way of successful problem solving
  - Problem-solving strategies still apply

You in the Situation

- Focus on what constructive action you can take
  - Focus on the future (what changes you want to see from here on)
  - Take responsibility for producing changes
- In contrast to:
  - Focus only on what the other person should do
  - Focus on the past (dwelling on problem)

Problem Solving Stance

- Get into the habit of seeing interpersonal difficulties as problems to be solved, as engaging the mind.
  - This is in contrast to reacting emotionally.
- “I don’t like this situation, how can I change it?”
  - Now you can invoke all the problem-solving machinery to generate potential solutions.

Example

The husband of a young wife would go out with one of his buddies “for an hour” and would come back two or three hours later. Resentment at being left alone builds up in the wife, and when the husband returns she starts scolding and yelling at him. This sequence, his staying out longer than he said and her yelling at him, would repeat itself two or three times a week.

Potential Solutions

- (When calm) Talk problem over.
  - Make him aware of your needs, etc.
- Rekindle romance (he stays home).
- Join him with friends sometimes.
- Have friends come over sometimes.
- Develop similar interests to why he goes out with friends.
- Find other things to do those nights for yourself.

Why the Problem Solving Stance?

- Why not react in anger if that is what the person deserves?
- You want to find a solution without bad “side effects”.
  - Collaborating mode, win-win.
  - Otherwise, risk increased conflict in future.

Example

George is a neat person. He has a good roommate, except for one thing. The roommate leaves dirty clothes around. George grumbles in silence for weeks. On the eve of a big date, George cleans up, and then the roommate comes in and leaves dirty cloths around. George blows up in anger.

Solutions

- Keeping quiet
  - Doesn’t solve the problem
- Getting angry
  - Might solve the immediate problem, has side effects
- Dumping roommate
  - Undesirable side effects

True goal: Neat apartment AND good relationship

Noise Example: Solutions

- Talk to the other person
  - How to do this effectively?
- Offer to buy him headphones
- Sleep with earplugs, add insulation
- Bring in rules enforcers
- Change rooms

Talking to the Other Person

- Talking to the other person often involves delivering criticism.
  - How can we do this effectively (solve problem without unwanted side effects)?
- Goal: Use “right speech”.

Presenting Yourself Well

- Make eye contact
  - In informal, conversational way.
- Use medium tone of voice.
- Humanize the situation.
  - Be friendly.
  - Use other person's name.
  - Be polite, use "please".
- Describe, not condemn:
  - "How I feel" more than "what you did".
  - Not "you are a slob", but "I have this problem with this behavior".

Presenting Yourself (cont)

- Goal: To get the other person to cooperate.
  - You want to be effective, not be right.
  - Have the other person see your rights, rather than just hear a demand.
- Anger creates Einstellung – avoid it.
- Visualize/rehearse the conversation.

Mediation

- A mediator is an (independent) third party who helps the involved parties negotiate a dispute.
- Why mediation can work:
  – Parties get to vent (as a first step).
  – Parties hear other side (perhaps for first time).
  – Parties hear the problem-solving approach as an alternative to conflict.

If you are asked to mediate:

- Don’t judge.
- Don’t dictate solution.
- Your job is to help parties find a solution.
- Adopt the problem-solving stance.
- Use “right speech”.
- Use lateral thinking, suggest creative alternatives.
- Present them as “what if” possibilities.

Special features

- Common metaphors for problem solving:
  - Moving forward
  - Making progress
- When you are stuck, how do you move forward?
- Hints can help… if you can get one
- How do you “give yourself” a hint?
- Look for special features in the problem.

Searching the Problem Space

```
   L E T S
+  W A V E
---------
 L A T E R
```

- Standard rules:
  - Letters consistently map to numbers
  - No leading zero (common use of numbers)
  - The numbers must work to add up correctly
- What is special here, to get us started?

Searching the Problem Space

```
    E A T
+ T H A T
---------
A P P L E
```

Standard rules:

- Letters consistently map to numbers
- No leading zero (common use of numbers)
- The numbers must work to add up correctly

What is special here, to get us started?

Another Addition Problem

```
  D O N A L D
+ G E R A L D
-------------
  R O B E R T
```

Given: D = 5.

Division Problems

```
     xx8xx
  ________
xxx|xxxxxxxx
    xxx
  ___
     xxxx
      xxx
    ____
       xxxx
       xxxx
       ____
```

Sudoku Puzzles

		1				8	9	
	2	7			9		5	
		4		8	2			
	6		9	2		1	4	
				5				
	9	8		6	1		3	
			2	1		4		
	1		7			3	6	
	7	9				2		

Problem

A man leaves his camp by traveling due north for 1 mile. He then makes a right turn (90 degrees) and travels due east for 1 mile. He makes another right turn and travels due south for 1 mile and finds himself precisely at the point he departed from, that is, back at his campsite. Where is the campsite located (or where on earth could such a sequence of events take place)?

This is searching the space of the solutions for special cases.

What are the special cases worth considering?

Go to Extremes

- Manipulate the problem space
- Look at extreme limits of the problem space.

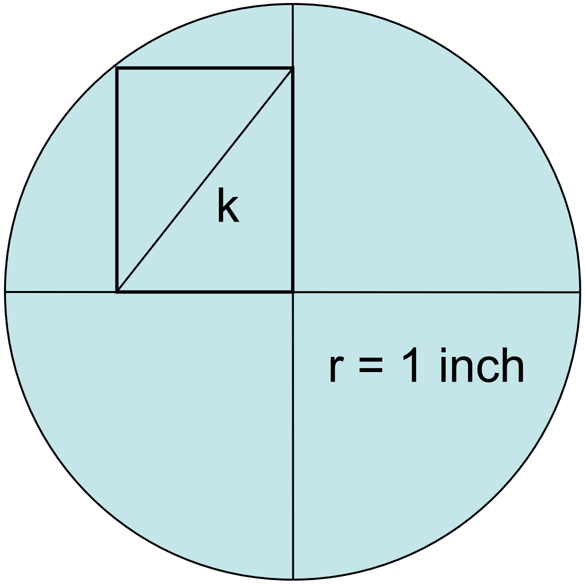
Example Problem

Two flagpoles are standing, each 100 feet tall. A 150-foot rope is strung from the top of one of the flagpoles to the top of the other and hangs freely between them. The lowest point of the rope is 25 feet above the ground. How far apart are the two flagpoles?

Hint: Start by drawing pictures.

Another example

- What is the length of k?
- Important fact: k remains the same no matter what rectangle is inscribed.



Another Example

You have a large, solid sphere of gold. A cylinder of space is “bored” through the center of this sphere, producing a ring. The length of that cylindrical line is 6 inches. You want to know how much gold you have left in the ring. Specifically, what is the volume of the ring? Note: For any sphere,

$$V = \pi D^3/6.$$

Simplify

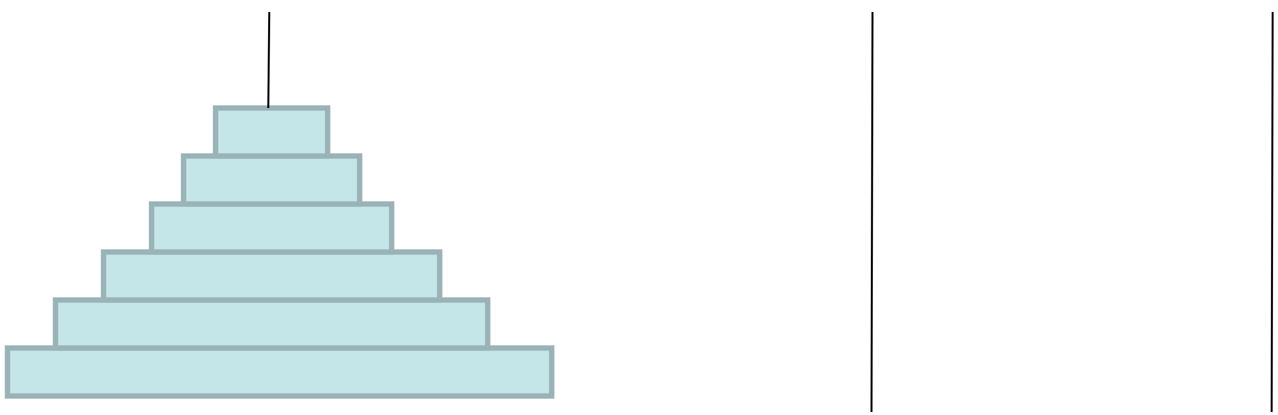
Take a number of several digits (say 7 or 8 digits). Reverse it and calculate the difference. Now if you tell me all but one of the digits in the answer (in any order), I can tell you the missing digit.

How can you go about figuring out the method?

- You can try some examples and look for a pattern.
- But if you do it on big numbers, it will be hard to figure out.

Towers of Hanoi

- Move one disk at a time
- No disk can sit on a smaller disk
- Get all disks from pole 1 to pole 3

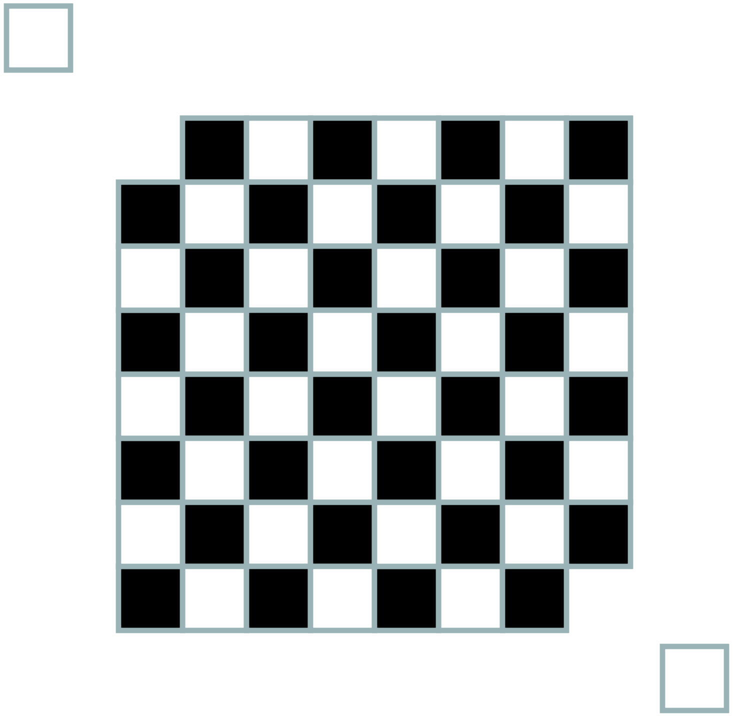


Chessboard Problem

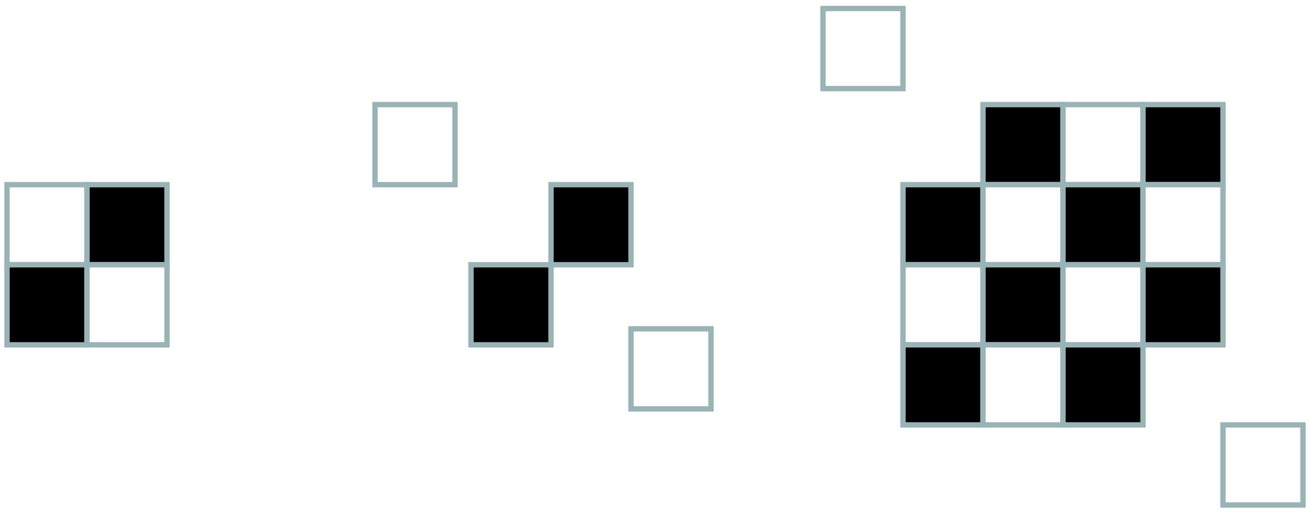
A domino covers two squares of a chessboard.

1. Can a chessboard be covered by dominos without any dominos sticking out?
2. Now, cut off the upper-left and lower-right corners of the chessboard. Can it still be covered by dominos completely?

Chessboard Visualization

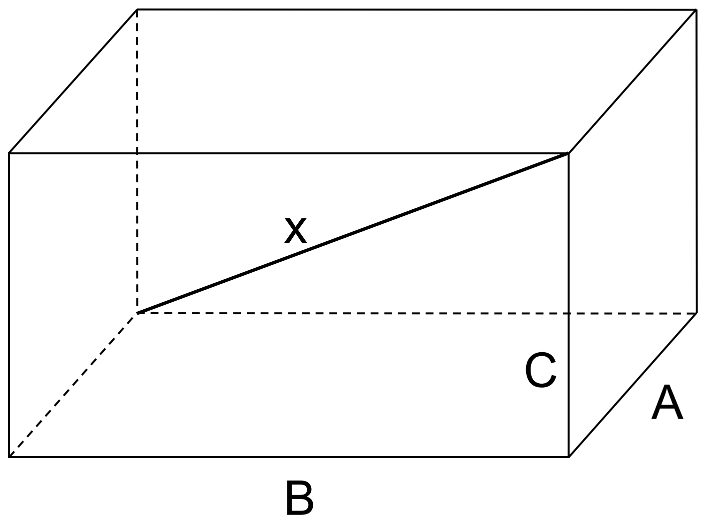


Simplify



Find the Diagonal

- You are given A, B, C. Calculate X.
- What is the simpler problem?
- How does it relate?



Coins Problem #1

You have 24 coins that look alike. With the exception of one counterfeit, they are all made of gold and weigh exactly the same. The “bad” coin is either heavier or lighter than the others, you do not know which. You also have available an old-fashioned balance scale. What is the minimum number of weighings you must make in order to locate the bad coin?

Coins Problem #2

You are given 10 stacks of what should be 10 gold pieces each. Each gold piece weighs two ounces. Unfortunately, one stack contains 10 counterfeits, each coin weighing only one ounce. You have a bathroom-type scale that reads out the weight of what is put on it. The problem: Determine the counterfeit stack with a single weighing.

Analysis of Trends and Patterns

- The goal is to identify the trend or pattern precisely
  - Don’t stop at simply identifying the “next step”.
  - Explicitly state what the pattern is that defined the next element in the series.

Sample Problems

- A B A C A D A E __ __ __
- 3 4 6 7 9 10 12 13 15 16 __ __ __
- 2 7 4 9 6 11 8 13 __ __ __
- 1 z 3 w 9 t 27 q 81 __ __ __
- JKLMNO JKLMON JKLOMN JKOLMN __ __

Jars Problem

You have 3 jars, of sizes 11 quarts, 9 quarts, and 4 quarts. You would like to use these jars to collect 6 quarts of water in one jar. How?

A	**B**	**C**	**Goal**	
11	9	4	6	A – B + C
21	127	4	98	
15	90	4	67	
14	163	25	99	
18	43	10	5	
9	43	6	22	
20	59	4	31	
14	36	8	6	
28	76	3	25	

Don’t be Blind

- For most problems, people use a relevant strategy from habit.
  - There’s an excellent reason for this: It usually works!!
- Sometimes, the habit strategy is a bad match for the problem.
- In this case, people can act like they are “blind” to the solution.
- Example: Water jar problem.

Einstellung

- “Einstellung” is the state of being “blind” or “tuned-in” to something.
- “Functional Fixedness”: People often fail to see alternate uses to an object once they assign it a role.
- People are fairly predictable in their susceptibility to functional blindness.
- Awareness of the problem helps to avoid it.
- This is real issue for students and in “real life”
  - Example: Debugging, algorithm design

Lateral Thinking

- "Vertical thinking" is sticking with the current approach, being rigid.
- "Lateral thinking" is coming at a problem from a different (perhaps nonstandard) direction.
- Often, just realizing that this should be done is enough to find a good solution (getting out of the old approach).
- Of course, it can be hard to tell when you are in the trap! It helps to have a "flexible" mindset.

Examples of Lateral Thinking

- Unsticking a car lock on a cold night
  - Approach 1: Heat the key
  - Approach 2: Unfreeze the lock (with alcohol)
- Need to iron a shirt, but no iron
  - Iron with something else (a frying pan)
- Sheep in front of the truck
  - Approach 1: Beep horn, try to push or scare sheep
  - Approach 2: Lead the sheep behind the truck

How to Facilitate Flexibility?

- Brainstorming (Chapter 6 in Fogler/LeBlanc)
  - Generate ideas
  - Usually done in groups
  - Don't judge – respect crude ideas
  - Quantity is important
- Brainstorming is a skill that can be developed
  - Skills are developed by practice

✧ A contrary view: D. Boyd and J. Goldenberg, "Inside the Box: A Proven System of Creativity for Breakthrough Results," Simon & Schuster, 2013.

The Intermediate Impossible

- For really hard problems
- Generate an impossible solution
- “Play with” that solution
  - Expand on it, modify it
- Thus, the “impossible” solution is an intermediate step to a feasible solution

Example Problems

- Unloading cargo ships takes a long time.
  - Unload at sea?
- New (taller) cargo ships cannot enter a port city due to a bridge.
  - Lower river?
- A factory dumps pollution into a river.
  - If the factory had to suffer from the pollution, they would be motivated to clean it up. So, put factory intake downstream from factory discharge?

Random Associations

- Pick an (interesting) word out of the dictionary.
- Let it stimulate your mind.
- Problem: Noise pollution
- Word: Anthracite
  - Comes from under ground
    - Put noise underground?
    - Put quiet places underground?
  - Black
    - Eyelids cover eyes… cover ears?

Analogies and Metaphors

- Many inventors take analogies from nature
  - Tunnels underwater: worms tunneling in wood
  - Microphone (for telephone) from the ear
  - Infection cause deduced from observing fermentation of wine
  - Spider nets lead to fishing nets

Sleep On It

- Passage of time can unstick many problems.
- The mind “incubates” the problem.
  - Perhaps works on problem unconsciously.
- Each of us has circumstances in which we are most creative:
  - lying in bed, taking a shower, waiting for an appointment.
  - Take advantage of this.
- Must give yourself time to solve the problem.
- Example: debugging a computer program.

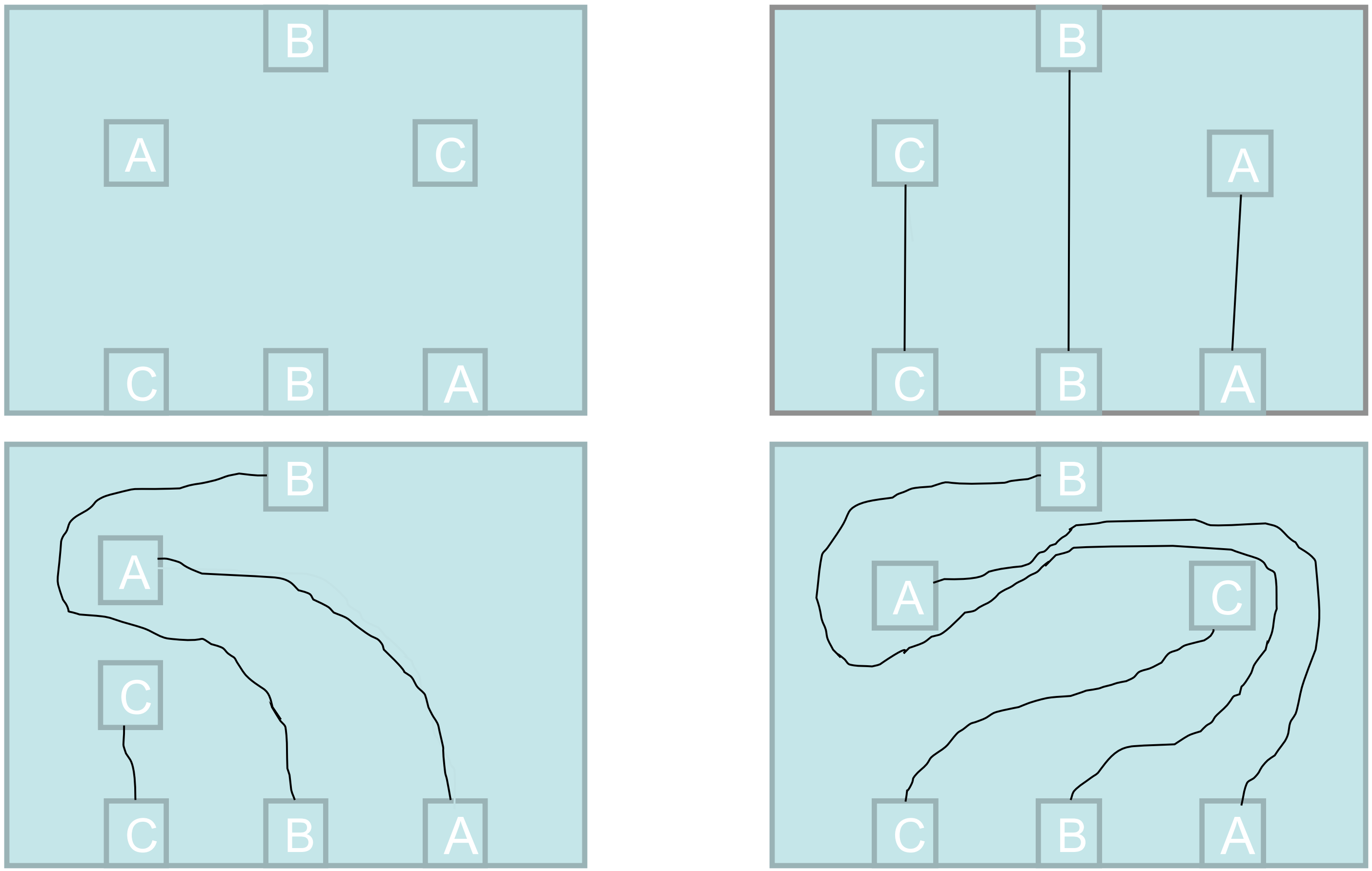
Sleep On It (cont)

- It gives you a chance to come at the problem with another approach
  - Does the solution occur to you?
  - Perhaps a new approach that immediately leads to the solution?
- Promotes (allows) lateral thinking

Heuristic: Wishful Thinking

- For some problems, you can get to a solution by:
  1. Solving a simpler form (wishful thinking: that the problem were simpler)
  2. Modifying the solution for the simpler form to become a solution for the original form

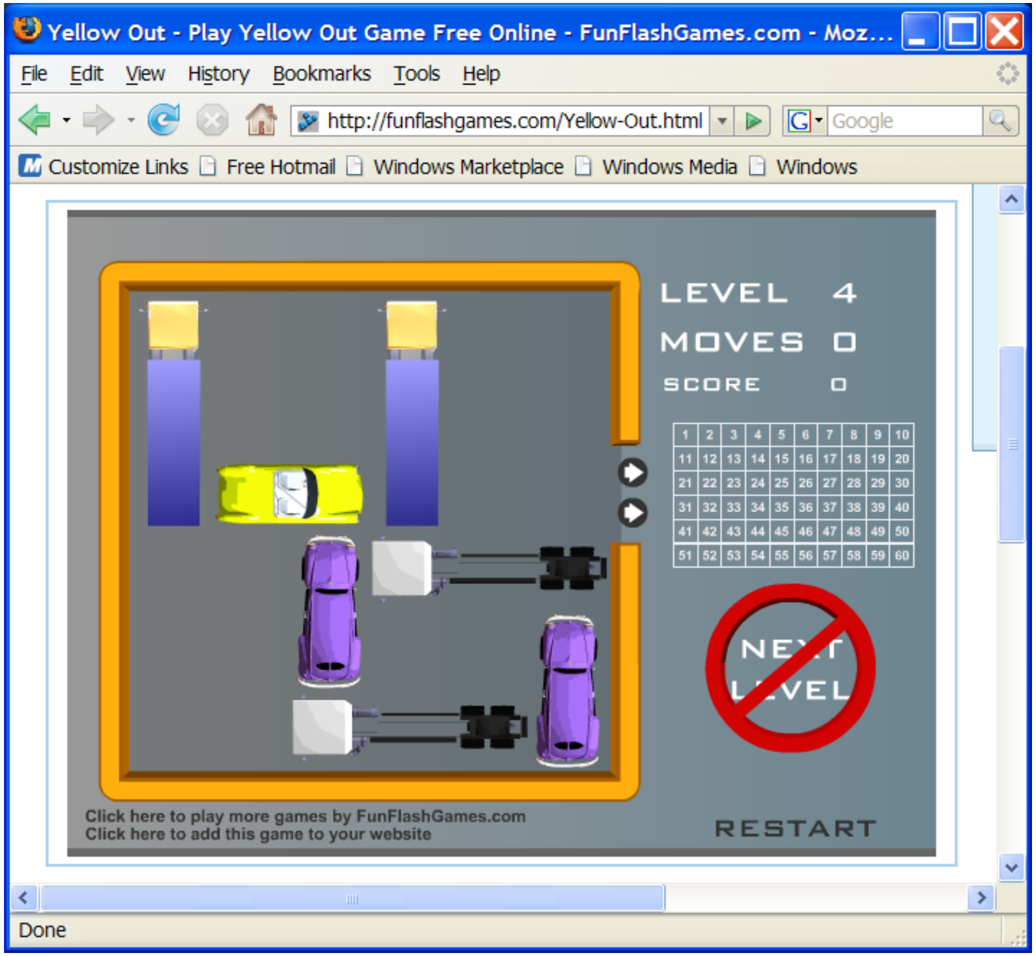
Wishful Thinking Example



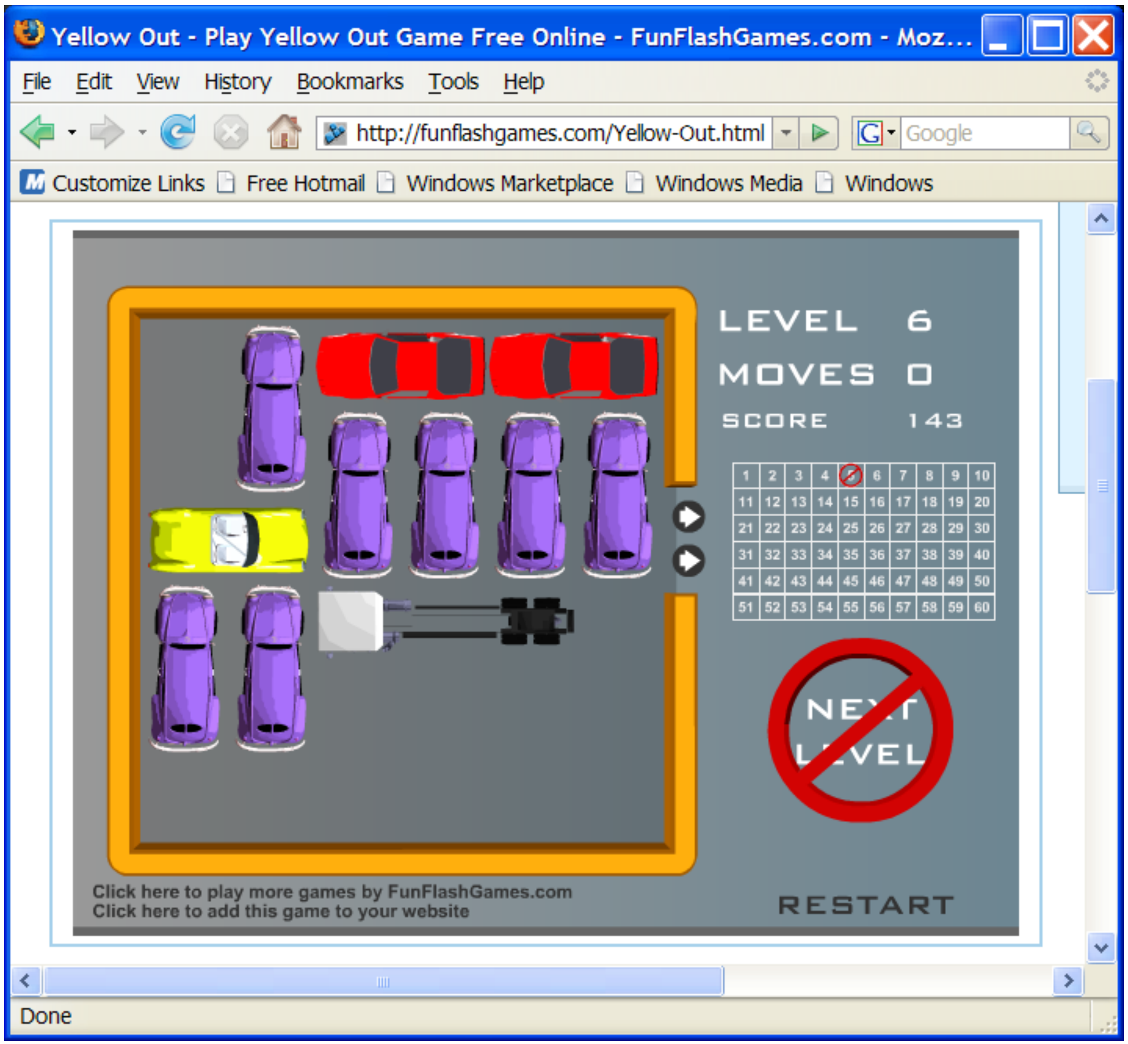
Penultimate Step

- Some problems can be viewed as moving from a start state to a goal state via a series of steps.
- If you can determine some intermediate step (I) on the path from start to goal, that simplifies the problem:
  - Move from Start to I
  - Move from I to Goal

Yellow-out Puzzle



Yellow-out (2)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Windows
LEVEL 6
MOVES 0
SCORE 143
1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
NEXT LEVEL
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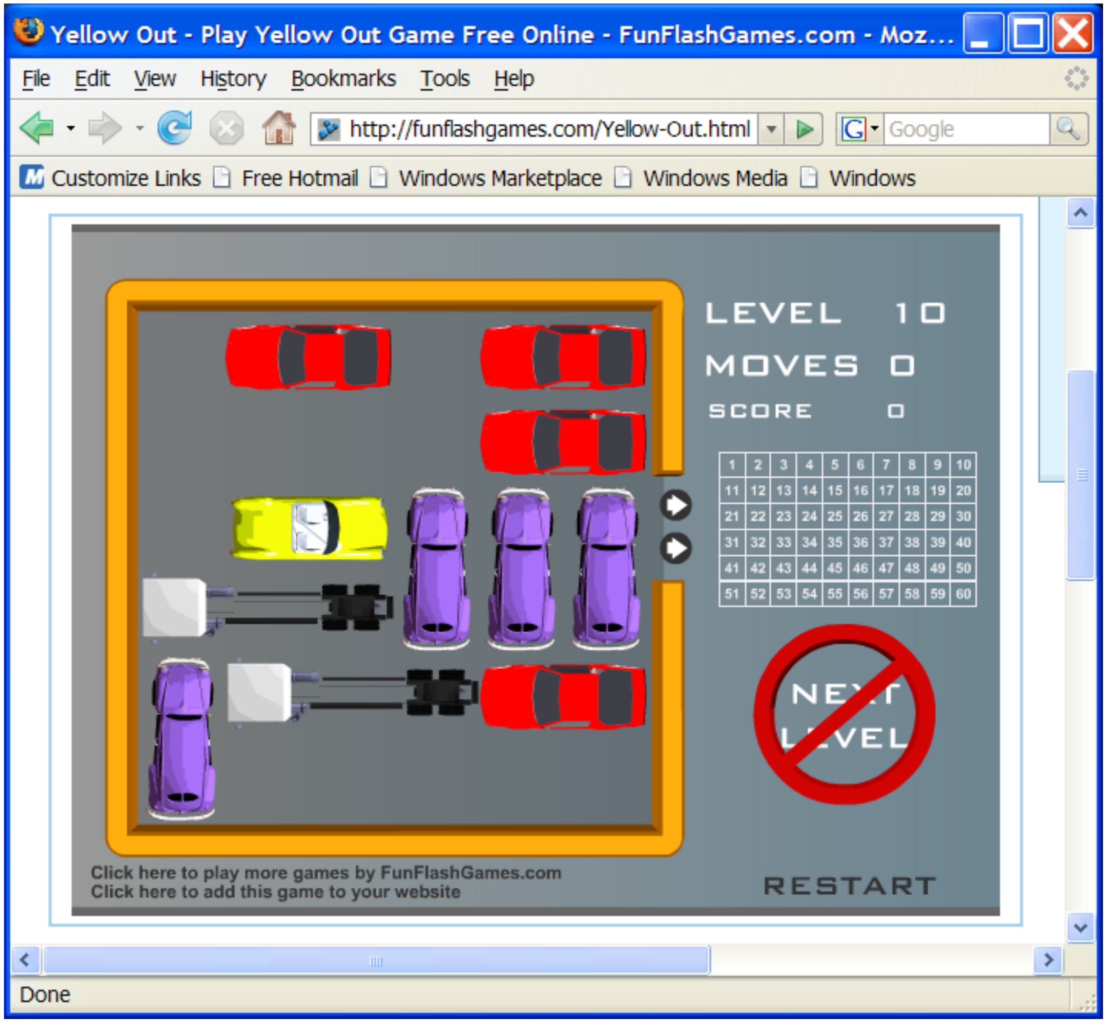
RESTART
Done

Yellow-out (3)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Windows Media
Windows
LEVEL 10
MOVES 0
SCORE 0
1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
NEXT LEVEL
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Click here to add this game to your website
RESTART
Done

Monks Problem

A monk climbs a mountain. He starts from the bottom at 8 AM and reaches the top at noon. He spends the rest of the day there. The next day, he leaves at 8 AM and goes back to the bottom along the same path. Prove that there is a time between 8AM and noon on each day that he is in the same place, at the same time, on both days.

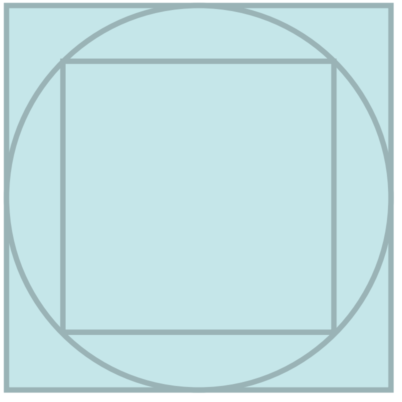
Stuck? Try drawing a picture.

Heuristic: Look for Symmetry

- If you find a symmetry, you might be able to exploit it
  - Symmetries give you “free” information, cut down on what to look at
  - Symmetries define an invariant
  - Symmetries indicate “special” points

Symmetry Problem

- What is the ratio of the areas of the two squares?



Symmetry Problem

Your cabin is two miles due north of a stream that runs east-west. Your grandmother's cabin is located 12 miles west and one mile north of your cabin. Every day, you go from your cabin to Grandma's, but first visit the stream (to get fresh water for Grandma). What is the length of the route with minimum distance?

Stuck? Draw a picture!

Symmetry Problem

What is the sum of the values 1 to 100?

Hint: Look for the symmetry!

The Pigeonhole Principle

If you have more pigeons than pigeonholes, when the pigeons fly into the holes at night, at least one hole has more than one pigeon.

Problem: Every point on the plane is colored either red or blue. Prove that no matter how the coloring is done, there must exist two points, exactly a mile apart, that are the same color.

Pigeonhole Problem

Given a unit square, show that if five points are placed anywhere inside or on this square, then two of them must be at most sqrt(2)/2 units apart.

Invariants

- An invariant is some aspect of a problem that does not change.
  - Similar to symmetry
  - Often a problem is easier to solve when you focus on the invariants

Motel Problem

Three women check into a hotel room with a rate of $27/night. They each give $10 to the porter, and ask her to bring back three dollars. The porter learns that the room is actually $25/night. She gives $25 to the desk clerk, and gives the guests $1 each without telling them the true rate. Thus the porter has kept $2, while each guest has spent $10-1=$9, a total of 2 + 3*9 = $29. What happened to the other dollar?

Invariant Problem

At first, a room is empty. Each minute, either one person enters or two people leave. After exactly 3^{1999} minutes, could the room contain $3^{1000} + 2$ people?

Invariant Problem

If 127 people play in a singles tennis tournament, prove that at the end of the tournament, the number of people who have played an odd number of games is even.

Deductive and Hypothetical Thinking

The day before yesterday you did not get home until yesterday; yesterday you did not get home until today. If today you do not get home until tomorrow, you will find that I have left yesterday.

- The mental reasoning needed here is fundamental in solving many problems.
  - Comprehend verbal statements.
  - Move in some dimension.
  - Think backward through a sequence to see where a movement began.

Simple Problems

Making a diagram helps with these.

- Suppose Valentine’s Day is 3 days after Friday. What day is Valentine’s day?
- Suppose Lincoln’s birthday is 4 days before Thursday. What day is Lincoln’s Birthday?
- Suppose Christmas is 2 days before Wednesday?
  - What day is Christmas?
  - What day is 4 days before Christmas?

Slightly Harder

- Saturday is 5 days before Labor day. What day is Labor day?
- Suppose Christmas is 2 days after Thursday. What day is Christmas?
- Suppose Thursday is 2 days after Christmas. What day is Christmas?

Deduction

- Today is Thursday. What is 2 days after tomorrow?
- Yesterday was Monday. What is 4 days after tomorrow?
- Today is Saturday. What is the day after 4 days before tomorrow?

Break It Down

Many math problems are solved by breaking them into parts. Your brain can only hold so much at once.

- Today is Monday. What is 1 day after 3 days before yesterday?
  - Use a diagram, and take it one step at a time.

Examples

- Yesterday was Tuesday. What is 2 days before 4 days after tomorrow?
- Tomorrow is Sunday. What is 2 days after 3 days before yesterday?
- Yesterday was Saturday. What is 4 days before 7 days after 2 days before today?
- Today is Monday. What is 3 days after 2 days before 6 days after 5 days after tomorrow?

Subtle Variations

- What is the difference between these two questions?
  - Today is Sunday. What is 3 days after today?
  - Sunday is 3 days after today. What is today?

More Examples

- Friday is 3 days before yesterday. What is tomorrow?
- Monday is 5 days before 2 days after yesterday. What is yesterday?
- Wednesday is 6 days before 2 days after tomorrow. What is tomorrow?

Mixed Problems

- Yesterday was Friday. What is the third letter in the day after tomorrow?
- If 6 days ago was Wednesday, what is the second letter after the second letter in 2 days after tomorrow?

Math-like Problems

- A man divides $1622.50 among four persons so that the first has $40 more than the second, the second $60 more than the third, and the third $87.50 more than the fourth. How much did the fourth person receive?
- A man bequeathes to his wife 1/3 of his estate; to his daughter, 1/5 of it; to his son, ½ of the daughter's share. He divides the remainder equally between a hospital and a public library. What part is received by the hospital?

Mathematical Word Problems

- A lot of word problems involve math.
  - That just means they involve (simple) numerical relationships.
  - Its all about setting up the relationships, not about the arithmetic.
- Process:
  - Be concerned about accuracy
  - Proceed step-by-step
  - Restate and subvocalize

Old Problem

Sally loaned $7 to Betty. But Sally borrowed $15 from Estella and $32 from Joan. Moreover, Joan owes $3 to Estella and $7 to Betty. One day the women got together at Betty’s house to straighten out their accounts. Which woman left with $18 more than she came with?

Hint: Make a diagram and use arrows to show which person has to return money to another person. Show the direction in which the money must be returned.

A Ratio Problem

A train can travel 10 miles in 4 minutes. How far will it travel in 14 minutes?

Alternative Solutions

- 14/4 = 3.5, so there are 3.5 (4-minute) units. The train goes 10 miles in each unit, so 3.5 x 10 = 35.
- Ratios: 10mi/4min = X mi/14min so (14)(10)/4 = X. X = 35mi.
- How many miles in one minute? 10mi/4min = 2.5mi/min. So in 14 minutes, 14min x 2.5mi/min = 35mi.
- Always write and check physical units!

Sample Problems

- Ted's weekly income is $100.00 less than double Gary's weekly income. If Ted makes $500.00 a week, what does Gary make?
- Paul makes $25.00 a week less than the sum of what Fred and Carl together make. Carl's weekly income would be triple Steven's if he made $50.00 more a week. Paul makes $285.00 a week and Steven makes $75.00 a week. How much does Fred make?

Investigation and Argument

- Solving a problem has two phases:
  - Investigation: Find a solution
  - Argument: Get the solution across to a “client”
- Too often, we only see the polished argument in a book
  - Hopefully this course helps you with investigation
- But you also have to be good at “argument”

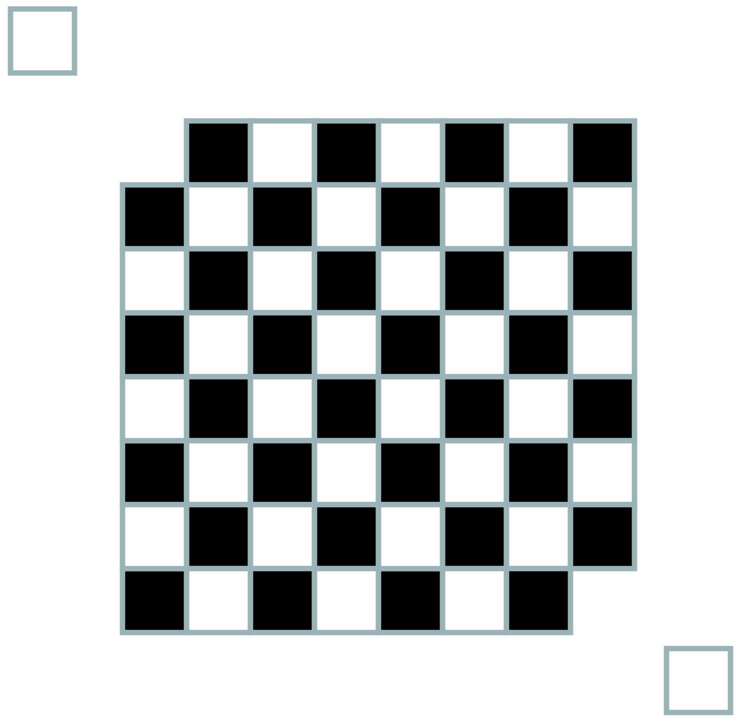
Argument

- Bad argument:
  - Coercing through force of will or personality
  - Irrespective of correctness
- Good argument:
  - Clear presentation
  - Logical progression of steps
  - Enough and not too much
  - Successful

When Do You "Argue"

- Anytime you write
  - Email
  - Letters
  - Tests
  - Homework
  - Proposals (anything where you ask someone to do something for you or give you something)
- Mechanisms:
  - Proofs
  - Essays, papers, books

Chessboard Problem



Problem: Completely tile defective chessboard with dominos

Chessboard Problem

Claim: Tiling the defective chessboard with dominos is impossible.

Proof?

Must be a convincing argument

First Proof Attempt

There are more black squares than white squares.
Therefore, tiling the defective chessboard with dominos is impossible.

Why is this not an adequate argument?

Second Proof Attempt

Every domino covers one black square and one white square.
The defective chessboard has two more black squares than white squares.
Therefore, any tiling with dominos will leave at least two black squares uncovered.
We conclude that tiling the defective chessboard with dominos is impossible.

Is this an adequate argument?

Form vs. Content

- There is a lot of competition for our attention.
- Usually you can’t get by with something that looks good but has no content.
- On the other hand, great content won’t be noticed without at least reasonable form.
- If things look bad, people will assume that the content is bad – and maybe look for trouble.
  - Especially important when turning in schoolwork!

Tests and Homework

- What is your goal?
  - To get the right answer? NO!
  - To get the best possible grade by:
    - Providing the right answer…
    - In a way that is legible, understandable, etc…
    - That convinces the grader that you got the right answer
  - The difference comes in recognizing how you will affect the grader

Tests and Homework (2)

- Formatting, legibility, presentation are crucial
  - Points in a list might be clearer than prose in a paragraph
  - Readable handwriting, clear writing matters!
  - Never say “**I think** 2 + 2 = 4”

Writing

- Speaking and writing skills make or break any professional career
  - It is a given that you won’t graduate without technical skills, so that will not be the crucial point of distinction.
- Good writing gives a competitive edge over poor writing
- How to improve your technical writing:
  - Write a lot
  - Fix five problems
  - Identify and focus on your purpose

Revising

- Repeated iterations of critical editing improves the writing product
- Eliminate useless words such as “very”
- Use the third person, never “I”
- Use present tense
- Avoid wishy-washy words: would, could, should, maybe
  - Be direct and confidence in your statements
  - Clean, direct writing is powerful

Purpose

- The purpose of writing is to convey something TO A READER
  - Your goal is to have an (appropriate) impact
  - Identify and focus on your audience
  - Tone matters. Example: email to a faculty member
- Keep your writing simple, clear

Reduce Cognitive Load

- The major goal of style is to reduce cognitive load on the reader
  - Simple notation
  - Clear layout
  - Syntactic consistency
  - Respect convention
  - Definitions near use
  - All things being equal, shorter is better
  - There can only be so many important things

Successful Technical Writing

- Direct
- Transparent
  - Does not call attention to itself
- Enjoyable
- Convincing
  - Answers skeptic's questions before they are asked
  - Includes necessary, but not extraneous, details
- Is easy (as possible) to understand

Computational Problem Solving

- Three pillars of science and engineering:
  - Theory
  - Experimentation
  - Computation (Simulation)
- Some problems are difficult to analyze analytically, but easy to simulate.
- Learn to “think computationally” to get results from simple simulations.
- Use computation/simulation to explore.
  - Tools like Matlab and Mathematica can help

Computational Example 1

- Birthday problem: Among a group of *n* people, what is the probability that two share a birthday?
  - This is related to hashing.
  - Can you determine this analytically?
  - How can you do this with simulation?

Algorithm #0

```
Apply[Plus, Table[ If[ Length[
Union[Table[Random[Integer,{1,365}],{n}]]]<n, 1, 0],
  numtrials] ]/numtrials
```

Algorithm #1

```
bool birthday(int count) {
  int myArray[365] = {0};
  for (int i=0; i<count; i++) {
    int pos = Random(365);
    if (myArray[pos] != 0)
      return true;
    else myArray[pos] = 1;
  }
  return false;
}
```

Issue: Must do it enough times to get meaningful statistics

Algorithm #2

```
double birthday(int count, int numtrials) {
  int myArray[365] = {0};
  int hits = 0;
  for (int trial=0; trial<numtrials; trial++) {
    for (int i=0; i<365; i++) myArray[i] = 0;
    for (int i=0; i<count; i++) {
      int pos = Random(365);
      if (myArray[pos] != 0)
        { hits++; break; }
      else myArray[pos] = 1;
    }
  }
    return (double)hits/(double)numtrials;
}
```

Computational Problem 2

- Analysis of hashing: What should we expect from a good hash function in terms of number of slots hit, length of chains?
  - Possible to analyze “ideal” performance analytically, but harder than simulating
  - Very hard or impossible to analyze performance of real hash functions analytically, but easy with simulation.

Things to Know

- Performance Measures:
  - How many slots were used (average)?
  - What is the minimum for slots used?
  - What is the longest chain ever?
  - What is the average for longest chain?
  - What is the expected cost?
- Issues:
  - Data Distribution
  - Fill factor
  - Table size (even/odd, power of two, etc)

Computational Example 3

- Do you know an algorithm to compute a square root?
- Assuming that you know how to multiply, can you think of a way to compute square roots?
- Guess/convergence testing is a fundamental concept for many numerical methods.

Algorithm

```
double squareRoot(double val) {
  double lower, upper;
  upper = val; lower = 1;
  if (val < 1) {lower = val; upper = 1}
  while ((upper - lower) > EPSILON) {
    double curr = (upper + lower)/2.0;
    if ((curr * curr) > val) upper = curr;
    else lower = curr;
  }
  return curr;
}
```

Computational Example 4

- Problem: Design a traffic light for an intersection
- You want to pick the best amount of time for each light phase
- Must allow every traffic direction access to the intersection in a reasonable length of time
- Goal: maximize the total traffic flow possible through the intersection
  - Other goals are possible
- Part of solution: traffic simulation

Traffic Simulation

- Consider all car directions, both from and to
- Traffic arrives at random, but typical, intervals
- Traffic light has a small number of states and timers
  - State transitions are programmed in light
- Simulation program runs simulated traffic through the intersection and measures the worst-case behavior
- Vary the state transitions to investigate different design possibilities

Problem Solving and Programming

- Design
  - Requires intense concentration
  - When is the best time to fix bugs?
- Testing
  - Requires a lot of skill, practice
  - How does problem solving relate to testing?

Debugging Example #1

A man who has had a heart attack goes every evening to a supervised exercise program. He handles the exercise well during the first 15 sessions, maintaining a heart rate at about 100 beats/minute. In the middle of the 16^{th} session, however, his heart rate suddenly shoots up to 130 beats/minutes. Although this may not be dangerous, nevertheless, the attendant has him stop exercising and calls the supervising doctor. The man is short of breath but otherwise feels fine. The change in heart rate appears to be his only symptom. What question(s) should the doctor ask?

Debugging Example #2

A man went to wash his face on awakening and found that there was no hot water. He knew to look for a special feature. He asked his wife whether she had done anything the day before near the boiler. Her response was in the negative. She added, however, “I didn’t have a chance to tell you, but the oil company sent a man yesterday to clean the furnace.” That certainly looked like a promising hint. A call to the oil company led to the solution of the problem.

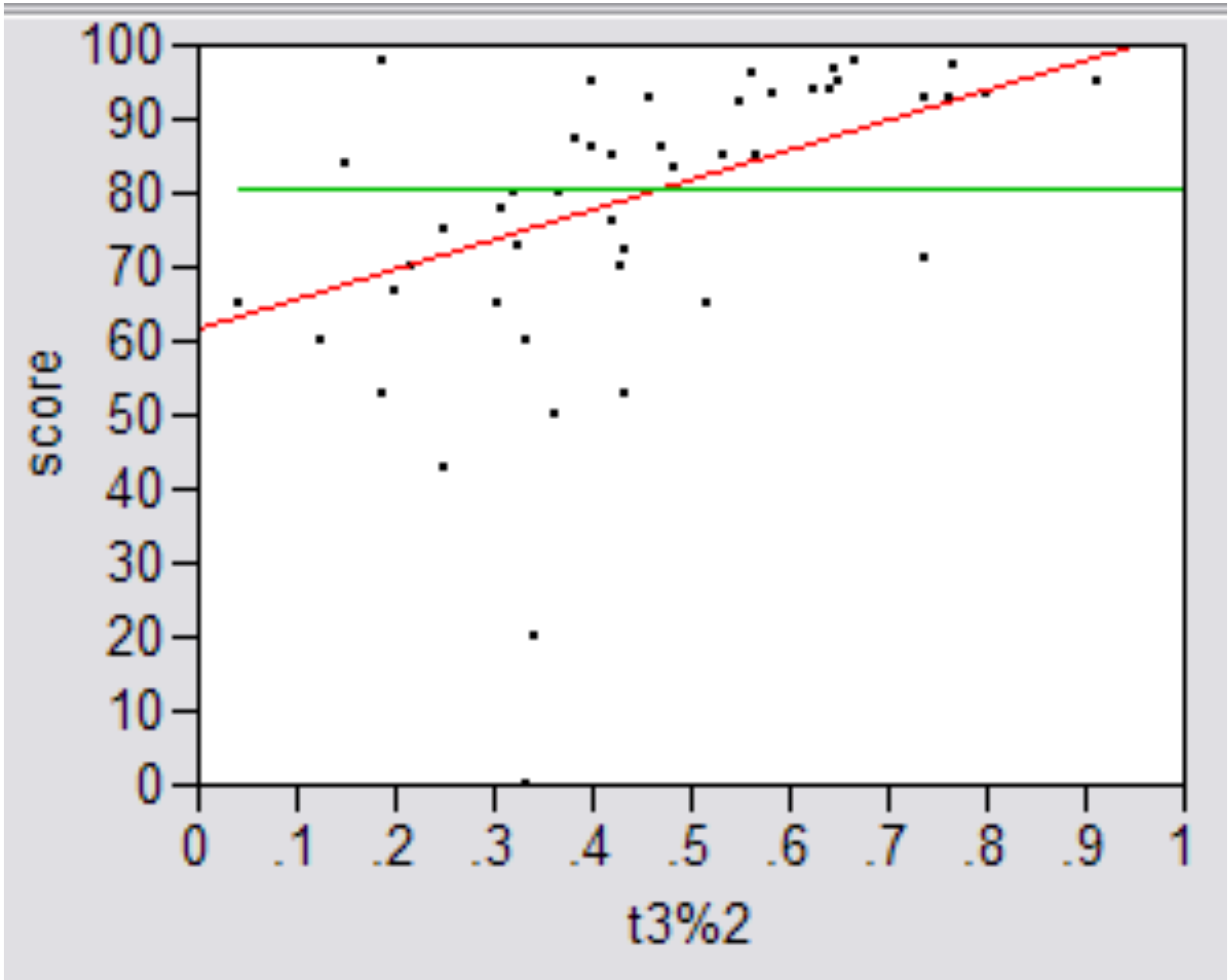
Debugging

- One of the hardest parts of programming.
- Strategy 1: Avoid bugs in the first place, by:
  - careful design (clean decomposition),
  - care with syntactic issues (layout, commenting).
- Strategy 2: Implement in a series of small steps, and test along the way.
  - This localizes new bugs to what changed in the program to introduce the bug.
- Finding bugs requires a disciplined, deductive approach.

Scheduling

- Managing large-scale projects involves significant efforts to plan and schedule activities.
  - It is human nature to work better toward intermediate milestones.
- The same concepts can/should be applied to mid-sized projects encountered in class.
  - For any project that needs more than a week of active work to complete, break into parts and design a schedule with milestones and deliverables.

Real Results



T3%2= percent of project completed before last week

Real Results

- Results were significant:
  - 90% of scores below median involved students who did less than 50% of the project prior to the last week.
  - Few did poorly who put in > 50% time early.
  - Some did well who didn’t put in >50% time early, but most who did well put in the early time.
- Correlations:
  - Strong correlation between early time and high score.
  - No correlation between time spent and score.
  - No correlation between % early time and total time.

What is the Mechanism?

- Correlations are not causal.
  - Do they behave that way because they are good, or does behaving that way make them good?
- Spreading projects over time allow the “sleep on it” heuristic to operate.
- Avoiding the “zombie” effect makes people more productive (and cuts time requirements).

Five Habits of Highly INeffective Programmers

1. Design with less than total focus
2. Disorganized code
   – Style, comments, design
3. Bite off more than you can chew
   – During implementation
4. Debug in a random walk
5. Program/debug in zombie mode
   – a.k.a. Don’t start early enough, don’t pace

7 Habits of Highly Effective People

(Covey)

1. Be proactive: Take initiative, seek new ideas.
2. Begin with the end in mind: Have a goal.
3. Put first things first: Prioritize, organize.
4. Think win/win: Seek mutual benefits.
5. Seek first to understand, then to be understood: Learn first, be adaptable.
6. Synergize: Make the whole greater than the parts.
7. Renewal: Physical, mental, spiritual, emotional.

Problem Solving in the Large

- In-the-small
  - There is an answer, the problem is to find it.
- In-the-large
  - Many possible solutions.
  - More complex problems -> more alternative solutions.
  - The goal is to pick the best solution.

Problem Solving Process

- Define the problem
- Generate solutions
- Analysis for deciding the course of action
- Implement the solution
- Evaluation

Problem Definition

- The first step is to define the “right problem”.
  - Often the hardest/most important step
- The “real problem” is often disguised.
- Symptoms vs. root problem
- Example 1:
  - Store had a rain forest health food mix.
  - It didn’t sell.
  - Perceived problem: overpriced.
  - Real problem: badly displayed.

Example: Flow Meter

- Flow meters in a chemical plant were being corroded and would leak.
- Perceived problem: “Find materials to make meter from that will not corrode”.
- After much effort, no such materials were found.
- Real problem: “Keep the flow meter from leaking”.
- Solution: Regularly replace (cheap) flow meters.

Example: Oil Recovery

- Oil company had underperforming oil field.
- Perceived problem: “Find ways to improve the oil recovery”.
- After years of effort, still no improvement.
- Eventually discovered that the estimates of oil in field were wrong.
- Real problem: “Learn why the well was not producing well”.

Example: Gas from Coal

- A coal-to-gas process was generating tar-like substances in pipes.
- Perceived problem: “Improve the solvents used to dissolve the coal to avoid the tar”.
- No solvent was found that worked.
- Real problem (generalize): “Determine why tar deposits are forming, and avoid them”.
- Solution: Increase velocity in pipes gives coal and solvent less time to react and scours pipes clean.

Example: Irrigation Dam

- Arid desert, but some plants grow.
- Solution: Design and build a dam to divert river water for irrigation.
- Very expensive dam was built.
- Result: High salt concentrations were dissolved, killing both old and new plants.
- How to avoid this outcome?

First Four Steps: Problem Definition

1. Collect and analyze information and data.
   - List every relevent thing you can think of.
   - Fill in missing gaps.
2. Talk with people familiar with the problem.
   - Look past the obvious.
   - Get clarifications when you don’t understand.
3. If at all possible, view the problem first hand.
4. Confirm all findings.

Example: Dead Fish

- Collecting data related to factory on river:
  - Fish kills below acceptable levels through July.
  - Fish kills above acceptable levels August 1, 15.
  - Toxins released on July 29 (but this level had not caused trouble before).
  - Water levels normal in July.
  - Water levels low in August.

Example: Hotel Elevator

- Hotel needs new elevators:
  - New shafts would cut rooms, etc.
  - Doorman suggested adding elevator to outside of building.

Example: Plastic Factory

- New factory generated defective plastic.
- Extensive analysis of design and materials detected no flaw.
- Eventually an engineer decided to look at the plant.
- A valve was set wrong, and no coolant reached the equipment.

Talk to “George”

- During collection stage, need to talk to the people familiar with the situation.
- Example: Equipment operators.
- Inspect the physical location if possible/appropriate.
- Most successful organizations have a person who both knows the facts on the ground and is a good “systems analyst”, and so can correlate the facts.

Example: Exploding Cabinet

- Electrical company installed an electrical cabinet at a customer's factory.
- The next day, the cabinet was energized, and exploded shortly after.
- The supervising engineer from the contractor went to inspect the site.
- He noticed that an unusual handle had been installed in the door of the cabinet.
- The factory maintenance staff had added the handle to allow access from their key.
- The handle had a metal rod protruding into the back.
- This metal rod caused a short circuit in the equipment.

Problem Definition

- Check problem statement with Socratic questioning (Critical Thinking Algorithm):
  1. Where did the problem originate?
  2. Who posed the problem statement? Your boss? Their boss? Colleague? Client?
  3. Can that person explain their reasoning?
  4. Are the reasoning and assumptions valid?
  5. Has that person considered different viewpoints?
  6. What are implications and consequences of assumptions?

Dead Fish 2

- Engineer asked to redesign plant with 1/10th the emissions.
- Why? Bad publicity from dead fish.
- Who? Upper management.
- Reliable assumption? Fish kills are correlated with low water. Toxins discharges are more concentrated then.
- Was enough toxin in the water? Not measured.

Dead Fish 2 (cont)

- Are there alternatives? Ask an expert. Fungi associated with heat and low water is suggested.
- Are fish dying elsewhere? Ask an expert (DNR), who says yes.
- Does the alternative pan out? Test the fish for fungi. Yes, it does.
- Conclusion: refitting the factory won’t help.

Present State vs. Desired State

- Define the present state.
- Define the desired state.
- Make sure both are precise.
- Make sure they match.

Example

- Situation: Too many bombers in WWII shot down. Many come back with bullet holes in similar spots.
- Perceived problem:
  - Many bullets penetrating aircraft.
  - Solution: reinforce damaged areas with thicker armor.
- Mismatch:
  - Present state: many bullets penetrating aircraft.
  - Desired state: fewer planes being shot down.
- Not a match because surviving planes have bullet holes.

Example (cont)

- New statements:
  - Present state: many bullets penetrating critical and noncritical areas.
  - Desired state: fewer bullets penetrating critical areas.
- These statements match.
- This focuses on the real problem.
- The original solution “fixed” something that wasn’t causing the real problem:
  - Planes with holes in noncritical areas were not the ones shot down, so don’t reinforce noncritical areas.

Dunker Diagram

- Trees in two dimensions
  1. Steps:
     - Goal
     - What to do
     - How to do it
  2. Desired state vs. make present state OK
- Example: Find a better job.
  1. Find a better job.
  2. Make present job OK.

Example: Teaching

- Problem: kindergarten teacher burned out from 25 years of teaching.

Quit teaching:

1. Find a new job:
    1. Office manager.
    2. Sales person.
2. Retire.

Make it OK not to quit:

1. More leisure time:
    1. teach alternate terms,
    2. teach half days.
2. Lower stress level:
    1. teach different grade,
    2. get more control over content.

Example: Cereal

- Situation: stale cereal in stores.
- Perceived problem: streamline the production process to get cereal to store shelves faster.

1. Get cereal to market faster.
   1. Build plants closer to market.
   2. Improve transportation.
      1. Hire faster trucks and race car drivers.
      2. Ignore speed limits.
      3. Use jet planes.

Cereal (cont)

1. Make it OK for cereal NOT to get to market faster.
    1. Stop making cereal.
    2. Make cereal stay fresher longer:
        1. add chemical to slow spoiling,
        2. make better boxes.
    3. Convince customers that stale cereal is OK.

Statement/Restatement

Original: Cereal is clearly not getting to market fast enough to retain freshness.

1. Read the sentence with emphasis on each of these words – what questions do they suggest?

- Cereal
- Getting
- Market
- Freshness

Statement/Restatement

Original: Cereal is clearly not getting to market fast enough to retain freshness.

2. Pick a term with a definition and replace the term with the definition, e.g.,
   - cereal -> breakfast food that comes in box,
   - market -> the place where it is sold,
   - retain freshness -> without getting stale.

- The change in emphasis makes us think about how we might change the box to prevent staleness, rather than thinking about speeding to market.

Statement/Restatement

Original: Cereal is clearly not getting to market fast enough to retain freshness.

3. Reverse: How can we make cereal get to market so slowly that it is never fresh?
   - This makes us think about how long we must retain freshness, and what controls it.

Statement/Restatement

Original: Cereal is clearly not getting to market fast enough to retain freshness

4. Change “every” to “some,” “always” to “sometimes,” etc.

- Cereal **sometimes** is not getting to market fast enough to retain freshness.
- Makes one think about things like why it is not **always** fresh, is it OK to occasionally not be fresh, etc.

Statement/Restatement

Original: Cereal is clearly not getting to market fast enough to retain freshness.

5. Challenge assumptions.
   - “Clearly” suggests an assumption.
   - Maybe cereal doesn’t get to store already stale?
   - Maybe the store holds it too long.
   - Maybe it is stale before it leaves the factory.

Statement/Restatement

Original: Cereal is clearly not getting to market fast enough to retain freshness.

6. Freshness is inversely proportional to the time since the cereal is baked: freshness = K/time .
   - Can we change K, the constant of proportionality? What does that depend on?
   - Packaging? Storage conditions? Type of cereal?
   - Change time? At factory? During shipping? Time to shelve? Shelf time?

K.T. Problem Analysis

- Useful for troubleshooting, where cause of problem is not known.
- Basic premise is that there is something that distinguishes what the problem IS from what it IS NOT.
  - The distinction column is the most important.

K.T. Problem Analysis

		IS	IS NOT	Distinction	Cause
What	Identify:	What is problem?	What is not problem?	What difference between is and is not?	What is possible cause?
Where	Locate:	Where is problem found?	Where is problem not found?	What difference in locations?	What cause?
When	Timing:	When does problem occur?	When does problem not occur?	What difference in timing?	What cause?
		When was it first observed?	When was it last observed?	What difference between 1st, last?	What cause?
Extent	Magnitude:	How far does problem extend?	How localized is problem?	What is the distinction?	What cause?
		How many units are affected?	How many not affected?	What is the distinction?	What cause?
		How much of any one unit is affected?	How much of any one unit is not affected?	What is the distinction?	What cause?

K.T. PA Example

On a new model of airplane, flight attendants develop rash on arms, hands, face (only those places). Only occurs on flights over water. Usually disappears after 24 hours. No problems on old planes over those routes. Does not affect all attendants on these flights, but same number of attendants get it on each flight. Those who get rash have no other ill effects. No measurable chemicals, etc., in cabin air.

K.T. PA Example

	IS	IS NOT	DISTINCTION
WHAT:	Rash	Other illness	External contact
WHEN:	New planes used	Old planes used	Different materials
WHERE:	Flights over water	Flights over land	Different crew procedures
EXTENT:	Face, hands, arms	Other parts	Something contacting face, hands and arms
	Only some attendants	All attendants	Crew duties

Picking a Technique

- Four strategies or procedures were discussed for defining the problem.
  - Which you actually use depends on the problem and your own style.
- You should consciously develop some process that addresses the major steps, which you use out of habit, to make sure that you do not end up solving the wrong problem.
  - Be proactive: think through whether the problem statement is correct before solving any problem.

Should We Solve It?

- If we are satisfied that we have defined the real problem, we next need to determine if it is worth solving.
  - Does it seem worth solving?
  - Has it been solved before (or is a suitable solution apparent)?
  - Are the necessary resources available?
  - Is there sufficient time?
- If any of these answers is “no”, can the constraints be changed?

Generating Solutions

- To succeed, ultimately you must:
  - define the correct problem,
  - select the best/acceptable solution for that problem.
- You can’t select an acceptable solution unless it gets on the list of potential solutions to be evaluated.
- You need an effective process for generating potential solution alternatives.

Mental Blocks (1)

1. Defining the problem too narrowly.
2. Attacking the symptoms and not the real problem.
3. Assuming there is only one right answer.
4. Getting “hooked” on an early solution alternative.
5. Getting “hooked” on a solution that almost works (but really doesn’t).
6. Being distracted by irrelevant information (mental dazzle).
7. Getting frustrated by lack of success.
8. Being too anxious to finish.
9. Defining the problem ambiguously.

Mental Blocks (2)

- There is a direct correlation between the time people spend “playing” with a problem and the diversity of the solutions generated.
- Sometimes problem solvers will not cross a perceived imaginary limit – some constraint formed in the mind of the solver---that does not exist in the problem statement.

Mental Blocks (3)

1. Stereotyping: functional fixedness (einstellung).
2. Limiting the problem unnecessarily.
3. Saturation or information overload.
4. Fear of risk taking.
5. Lack of appetite for chaos.
6. Judging rather than generating ideas.
7. Lack of challenge.
8. Inability to incubate.

Sources of blocks: culture, environment, inability to express, inflexible/inadequate problem solving skills.

Blockbusting Problems/Solutions

1. Negative Attitude: Attitude Adjustment
   – List positives, focus on opportunity instead of risk.
2. Fear of Failure: Risk Taking
   – Define the risks and how to deal with them.
3. Following Rules: Breaking Rules
   – Try new things, new foods, new places.
4. Overreliance on Logic: Internal Creative Climate
   – Let imagination work, play with it.
5. Believing Not Creative: Creative Belief
   – Ask “what if,” daydream, make analogies.

Improving Creative Abilities

- Keep track of ideas (write them down immediately).
- Pose new questions to yourself every day.
- Keep abreast of your field.
- Learn about things outside your specialty.
- Avoid rigid, set patterns of doing things.
- Be open and receptive to new ideas.
- Be alert in your observations.
- Adopt a risk-taking attitude.
- Keep your sense of humor.
- Engage in creative hobbies.
- Have courage and self confidence.
- Learn to know and understand yourself.

Generating Solutions

- Brainstorming
- Futuring
- Analogy and Cross-fertilization

Generating Solutions: Brainstorming

- Free Association Phase
  - Unstructured.
  - Generate lots of ideas.
  - Ideas flow freely for awhile, then taper off.
  - How to generate more ideas?
- Vertical Thinking
- Lateral Thinking

Vertical Thinking

A more structured approach to generating new ideas as part of brainstorming.

- Adapt: How can we use this?
- Modify: What changes can we make?
- Magnify: Add something? Make stronger, longer, etc.?
- Contract: Split up? Lighten?
- Rearrange: Interchange, reorganize?
- Combine: Compromise? Blend?

Lateral Thinking

Random Stimulation

- Select a word from the dictionary or a list of “stimulating” words.

Other People’s Views (OPV)

- Imagine yourself in other roles.

Futuring

Ask leading/stimulating questions, ignore technical feasibility (aka wishful thinking).

- What are the characteristics of an ideal solution?
- What currently existing problem, if solved, would make our lives/jobs easier, or make a difference?

Futuring Example

Cheese/yogurt factory generates acidic waste byproducts. Traditional approach is to “treat” the waste so that it can be discharged.

Futuring: Imagine a successful plant with no waste. All such “waste” has a useful purpose.

- Protein: Food additives/supplements.
- Sugar: Ferment for Ethanol.
- Solid waste: De-icing compound, construction material.

Real problem: What to do with waste?

Fishbone Diagrams

- Used to organize and record brainstorming session.
- Backbone is the problem to solve.
- Categorize solutions. Each is a diagonal spur.
- List the solutions on each spur, perhaps generate subspurs.

How to use Cars in Playgrounds

Painting

- Let kids paint graffiti on cars.
- Paint targets and throw balls at them.
- Paint as something (wagon) for play.

Whole Car

- Make teeter-totter (upside down).
- Turn into a go-cart.
- Let kids drive it.

Parts

- Use seats as swings.

Analogy

1. State the problem.
2. Generate analogies (the problem is like...).
3. Solve the analogy.
4. Transfer solution to problem.

Cross-Fertilization

Much of science is done by combining ideas from different fields.

Imagine a meeting between pairs such as:

- beautician and college professor,
- police officer and software programmer,
- automobile mechanic and insurance salesman,
- banker and gardener,
- choreographer and air traffic controller,
- maitre d' and pastor.

Incubating Ideas

- Like “sleep on it”: no time to consider possibilities leads you to grab an early, sub-optimal solution.
- In contrast, including some “incubation” time can help the process.

Deciding the Course of Action

Kepner-Tregoe (K.T.) situation appraisal approach:

- K.T. problem analysis: (discussed before)
  - Past: What is at fault?
- K.T. decision analysis:
  - Present: How to correct the fault?
- K.T. potential problem analysis:
  - Future: How to prevent future faults?

K.T. Situation Appraisal

- For prioritizing multiple problems.
- Make a list of all problems.
- For each, assign scores (H, M, L).
  - Timing: How urgent?
  - Trend: What is happening over time?
  - Impact: How serious is problem?
  - Which K.T. analysis? (PA, DA, PPA)

SA Example: Store Manager

Major Concern	Subconcern	Timing	Trend	Impact	Process
Space	Unopened boxes				
	20 new desks				
Personnel	Employee morale				
Finances	Money owed				
	Money due				
Quality	Scratched desk				

K.T. Decision Analysis

1. Write a concise decision statement about what it is we want to decide.
   – Use first four problem-solving steps to gather information.
2. Specify objectives of the decision, and divide into **musts** and **wants.**
3. Evaluate each alternative against the musts:
   – “go” vs. “no go”.
4. Give a weight (1-10) for each want.
   – Pairwise comparison can help with relative weights.
5. Score each alternative.

K.T. DA Example

MUSTS		Paint Right		New Spray		Gun Ho
Adequate flow control		Go		Go		No go
Acceptable appearance		Go		Go		Go
WANTS	**weight**	**Rating**	**Score**	**Rating**	**Score**	
Easy service	7	2	14	9	63	No go
Low cost	4	3	12	7	28	
Durability	6	8	48	6	36	
Experience	4	9	36	2	8	
Total			110		135	

K.T. Potential Problem Analysis

- Analyze potential solutions to see if there are potential problems that could arise.
- Ones not analyzed in prior steps.
- Particularly appropriate for analyzing safety issues.

K.T. PPA Example: Buying Car

Problem	Possible Cause	Preventive Action	Contingency Plan
Improper alignment	Car in accident	Check alignment	Don’t buy
Body condition	Car in accident; body rusted out	Inspect body for rust	Offer lower price
	Car in flood	Check for mold/ hidden rust	Offer lower price
Suspension problems	Hard use, poor maintenance	Check tires	Require fixes
Leaking fluids	Poor maintenance	Inspect	Require fixes
Odometer incorrect	Tampering/broken	Look for signs, check title	Offer lower price
Car ready to fall apart	Poor maintenance	Look for signs	Don’t buy

Implementing Solution

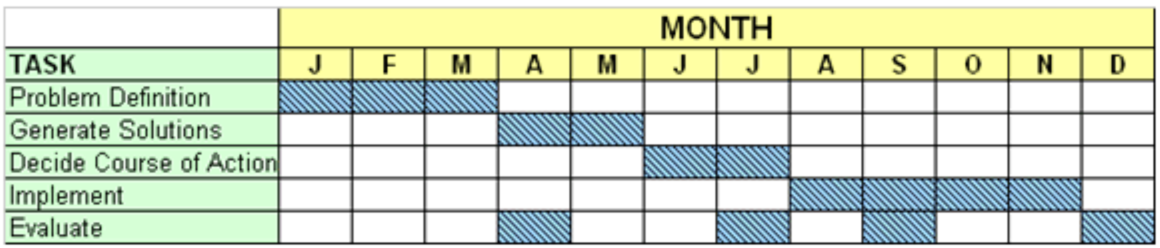
- Approval
- Planning
- Carry through
- Follow up

Approval

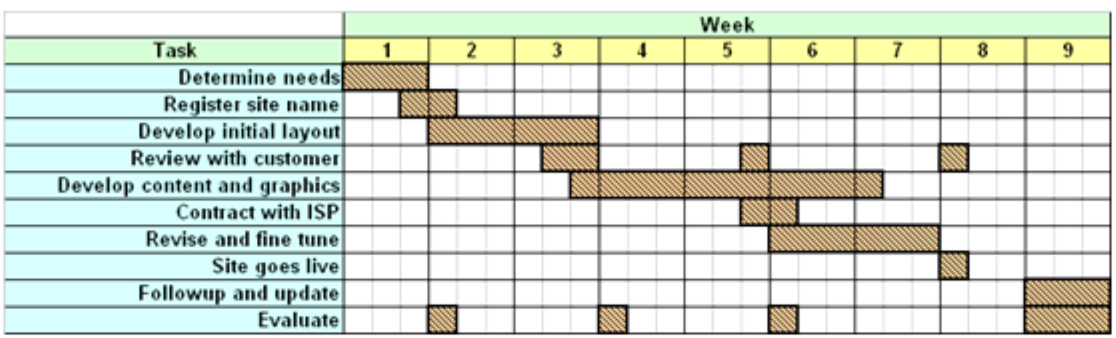
- From authorities or clients
- Make a proposal
  - All of the presentation issues apply
  - Must especially focus on the client's goals

Planning Techniques (1)

The Gantt Chart



Web Site Development Gantt Chart



The Gantt Chart graphically shows the progression of work required to complete the project.

- Gantt chart for allocating resources, time

Planning Techniques (2)

Deployment Chart for the Website Development Project

Task	Team Member		
	Melinda	John	Web Programmer
Determine needs			
Register site name			
Develop initial layout			
Review with customer			
Develop content and graphics			
Contract with ISP			
Revise and fine tune			
Site goes live			
Followup and update			
Evaluate			

- Deployment chart
- Critical path analysis
- Allocating/budgeting resources

Carry Through

- Actual management of the implementation.
  - Estimate what finished project will look like.
  - Ensure coordination of tasks and personnel.
  - Steadily monitor Gantt Chart, etc.
  - Evaluate each completed step along the way.
  - Continue to learn about solution.
  - Continue to test assumptions about solution.
  - Test the limits of the solution.
  - Carefully plan test simulations.

Follow Up

- Follow Up
  - This refers to monitoring the implementation process and adjusting as necessary.
  - Following the plan?
  - Proceeding on schedule?
  - Staying within budget?
  - Maintaining quality?
  - Relevant to (original? changing?) problem.

Evaluation

- Evaluation should be an ongoing process throughout life of the project.
- Each phase of the project should have a review to verify that goals of the phase were accomplished.
- This might cause adjustments to future plans.
- For each decision, carry out a PPA before implementing the solution.

Evaluation Checklist

- Have you challenged the information and assumptions?
- Does the solution solve the real problem?
- Is the problem permanently solved? Or is this a patch?
- Does the solution have an impact on the problem?
- Have all consequences of the solution been considered?
- Have you argued both sides, positive and negative?
- Has the solution accomplished all that it could?
- Is the solution economically efficient and justifiable?
- Have the “customers” bought in?
- Does solution cause problems (environmental, safety)?

Ethics Checklist

- Is it legal? Does it violate the law, or organizational policy?
- Is it balanced? Is it fair to all concerned in short and long term? Is it a win-win solution?
- How will it make me feel about myself? Will it make me proud? How would I feel if it were published in the newspaper? If my family knew?

Multidimensional Problems

- Some problems ask to find an optimal solution.
  - Ex: Buy the best computer under $1000.
- There may be multiple factors, and they may interact.
  - Ex: CPU, memory, disk, graphics card.
- The goal can be thought of as finding the best point in a multidimensional space, where each point has a value.
  - Ex: For some combination of CPU, memory size, disk drive, and graphics card, what is the performance?
  - Constraint: Cost < $1000.

Experimental Design

- There might be so many factors, and possible values for the factors, that you can’t afford to test every combination.
- Experimental design refers to selecting specific combinations of factor values to test.
- Full factorial design with m levels in p dimesions requires m^p experiments.
- Central composite design in p dimensions requires $2^p + 2p + 1$ experiments.
- Other designs include Latin hypercube, D-optimal, orthogonal, etc.

Statistics

- See the statistics notes in PSNotes_math.pdf.
- Often you wish to get a measure of some performance metric from a random sample.
  - Ex: Mean height of college students (population).
- Any given sample statistic is not the true population statistic.
  - It is a random variable with some distribution.
  - You need to figure out how to get a reasonable estimate for the population statistic.

Estimating Issues

- Sample the population
  - How to sample?
  - How many to sample?
  - How confident are you about the result?
- Hypothesis testing
  - Is one mean bigger than another?
  - With what probability?
- These are the things that a statistics course attempts to teach you.

Making an Argument

- The goal of communication is to achieve the desired affect on the target audience.
- Often we want to convince the audience of something:
  - answers on an exam,
  - making a proposal,
  - letter to the editor.
- The goal is not just to <u>be</u> right.
- The goal is to convince the audience that we <u>are</u> right.

Investigation and Argument

- How can we be convincing?
  - Need to be right (investigation/solution).
  - Need to present it right (argument).
- Part of good communication is to reduce cognitive load on the audience.
- Good technical writing is essentially about making clear, logical arguments.
- Following standard presentation forms can help.
  - Conventions in reasoning.
  - Proof forms.

Mathematical Proof

- "Mathematical" proofs often follow one of several standard forms.
  - These forms have proved useful for structuring ideas.
  - Following a conventional form reduces cognitive load on the reader.

Deduction (Direct Proof)

- If P, then Q.
- P implies Q.
- Contrapositive: (not Q) implies (not P).
- All the above mean: not (P and (not Q)).
- Sometimes you can break this down into parts:
  - assumptions imply first step;
  - first step implies second step; …
  - truth of the penultimate step implies the conclusion.

Reasoning Chains

- Many systems work by chaining a series of steps
  - Symbolic logic
  - Geometry proofs
  - Calculus derivatives and integrals

Proof by Contradiction

- Want to prove X.
- Assume that X is false.
- Show that this assumption leads to a logical contradiction.
- Since the assumption must be false, X must be true.

Contradiction Example 1

Prove that there is no largest integer

- Assume that there is a largest integer, B.
- Consider C = B + 1.
- C is an integer (the sum of two integers).
- C > B.
- Thus, B is not the largest integer, a contradiction.
- The only flaw in the reasoning was the assumption that there exists B, the largest integer.
- Therefore, there is no largest integer.
- This proof is in Euclid's <u>Elements</u>.

Contradiction Example 2

Prove that $\sqrt{2}$ is irrational.

- Suppose $\sqrt{2}$ is rational.
- Then $\sqrt{2} = a/b$ for a and b integers with the greatest common divisor GCD(a,b) = 1.
- Since $2b^2 = a^2$, a^2 is even (so a is even).
- So $a = 2t$, yielding $2b^2 = a^2 = 4t^2$.
- So $b^2 = 2t^2$, making b even.
- Thus GCD(a,b) >= 2, a contradiction.
- Therefore $\sqrt{2} = a/b$ is not possible.

Mathematical Induction

- To prove by induction, must show two things:
  - **Base case**: the result is true for some integer k.
  - **Induction step**: Being true for n-1 >= k implies that it is true also for n.
- Often easy to prove base case (k=0 or k=1).
- Might or might not be easy to prove the induction step.
  - Note that we are proving an implication:
    S(n-1) implies S(n) .

Induction Hypothesis

- The key to induction is the induction hypothesis.
- We assume S(n-1) is true.
- This gives us material to work with.
- It is also what confuses people most.
- Recall the truth table for an implication. It tells us that we only care about the situation where the assumption is true.

A	B	A implies B
T	T	T
T	F	F
F	T	T
F	F	T

Induction Example

Call S(n) the sum of the first n integers. Prove that S(n) = n(n+1)/2.

- **Base case**: S(1) = 1(1+1)/2 = 1, which is true.
- **Induction hypothesis**: S(n-1) = (n-1)n/2 for n-1 >= 1.
- **Induction step**: Use the induction hypothesis.
  - S(n) = S(n-1) + n .
  - S(n) = (n-1)n/2 + n = (n^2 – n + 2n)/2 = n(n+1)/2 .
- Therefore, the theorem is proved by mathematical induction.

Induction Example

- 2-cent and 5-cent stamps can be used to form any value $n \geq 4$.
- **Base case**: $2 + 2 = 4$.
- **Induction hypothesis**: Assume true for any value n-1 >= 4.
- **Induction step (S(n-1) implies S(n))**:
  - Case i: A 5-cent stamp is replaced with 3 2-cent stamps.
  - Case ii: Two 2-cent stamps are replaced with a 5-cent stamp.
- Therefore, the theorem is proved by induction.

Induction and Recursion

- Induction and recursion are similar.
- If you are comfortable with one, should quickly be able to grasp the other.
- Both have a base case.
- Both use the assumption that subproblems are true/solvable:
  - recursion makes a recursive call;
  - induction uses the induction hypothesis.
- A recursive function's primary work is converting solutions to subproblems into the full solution;
  - this is the same as the induction step.

Forms of Induction

- “Standard” induction: S(n-1) implies S(n).
- “Strong” induction: S(k) to S(n-1) implies S(n).
  - Strong induction gives us a stronger induction hypothesis.
  - The induction hypothesis is free material to work with.

Strong Induction Example

- For *n*>1, *n* is divisible by some prime number.
- **Base case**: choose n=2. 2 is divisible by the prime number 2.
- Induction hypothesis: any value *a*, 2 <= *a* < *n*, is divisible by a prime number.
- Induction step for value *n*:
  - Case 1: If *n* is prime, *n* is divisible by *n*.
  - Case 2: If *n* is composite, *n* = *a* x *b* for *a*, *b* integers >=2 and less than *n*. The induction hypothesis says that *a* is divisible by a prime number, so *n* is divisible by a prime number.

Guess and Test

- One approach to problem solving is to guess an answer and then test it.
- When finding closed forms for summations, can guess a solution and then test with induction (but using the fundamental theorem of difference calculus is better).
- Induction can test a hypothesis, but doesn't help to generate a hypothesis.