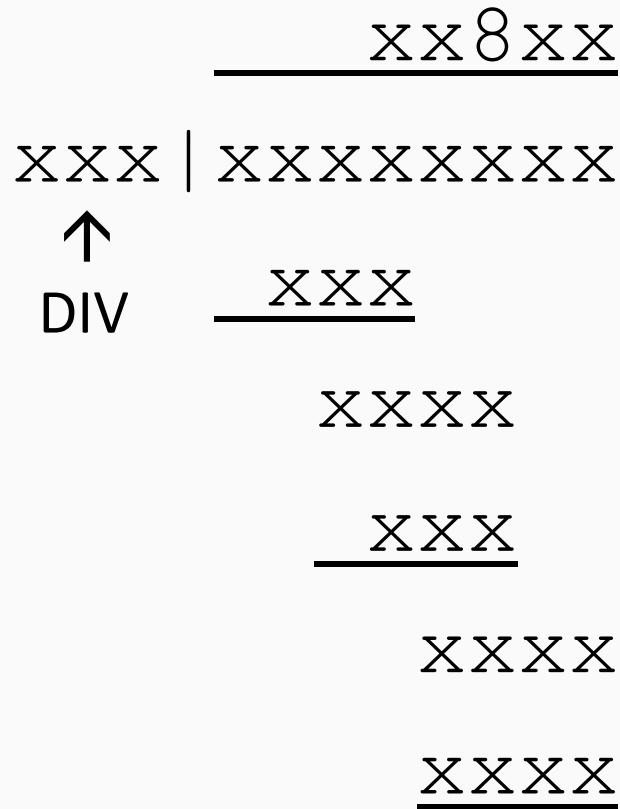


Division Problem

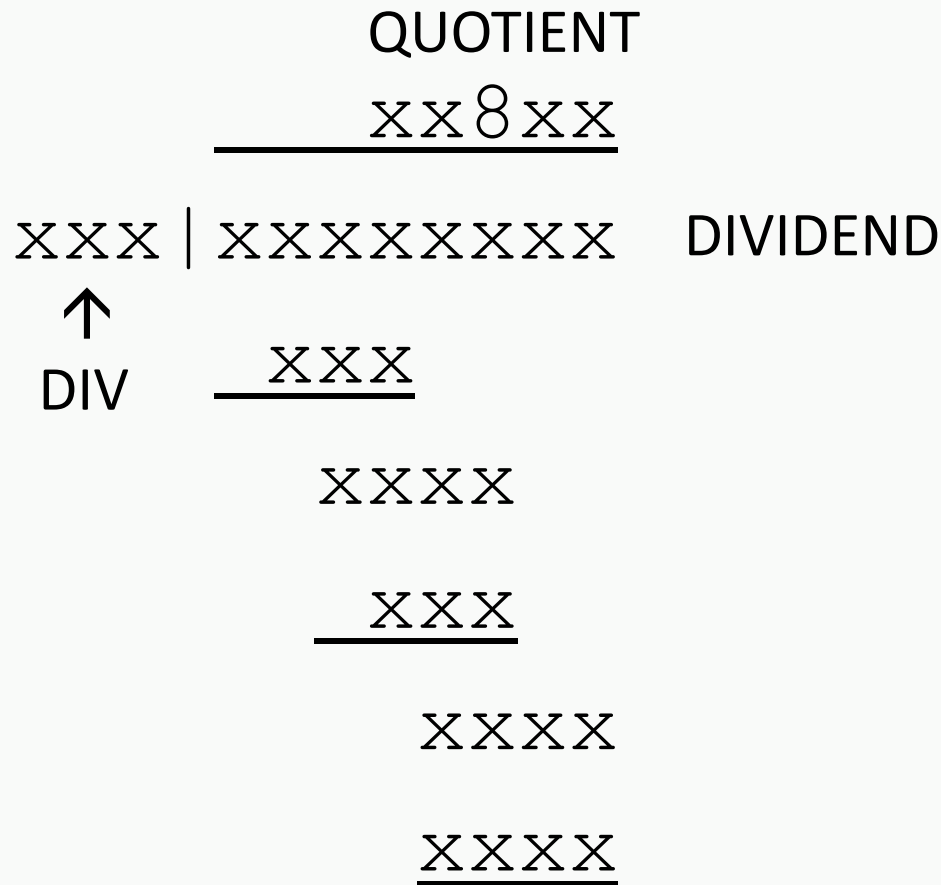
```
            xx8xx
xxx | xxxxxxxxx
      xxx
      xxxxx
      xxx
      xxxxx
      xxxxx
```



Substitute a digit between 0-9 for each X. Initial digits are never 0.

Look for special features.

Division Problem: Getting Started

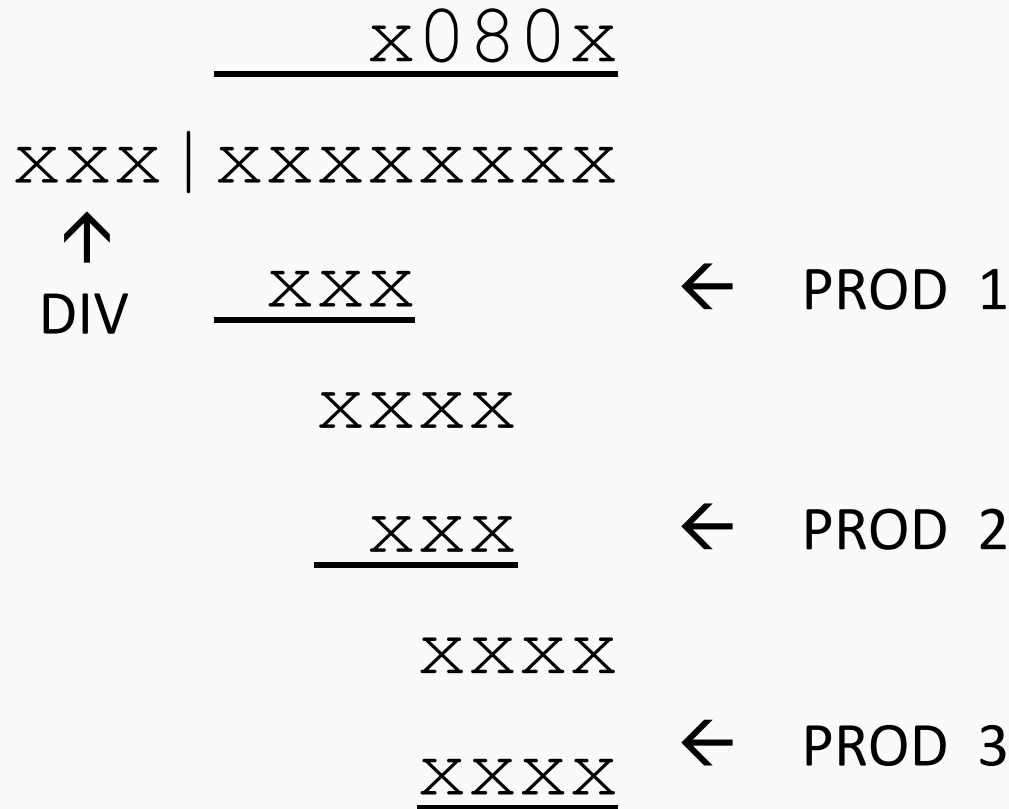


Special features:

PROD 1 not placed under the first 3 digits?

We "skip" a column between the first digit of the quotient and the 8; that means the intervening digit must be a 0.

Likewise, the next-to-last digit in the quotient must be a 0.



Special features:

PROD 2 result of known multiplier: 8

Multiplying DIV by 8 yields a 3 digit number.

Thus DIV must be a small number in the range 100-124 (since $125 * 8 = 1000$).

So PROD 2 is a number between 800-992.

Division Problem: Special Features

PROD	PROD
2	2
XXXX	XXXX
<u>-8XX</u>	<u>-9XX</u>
XX	XX

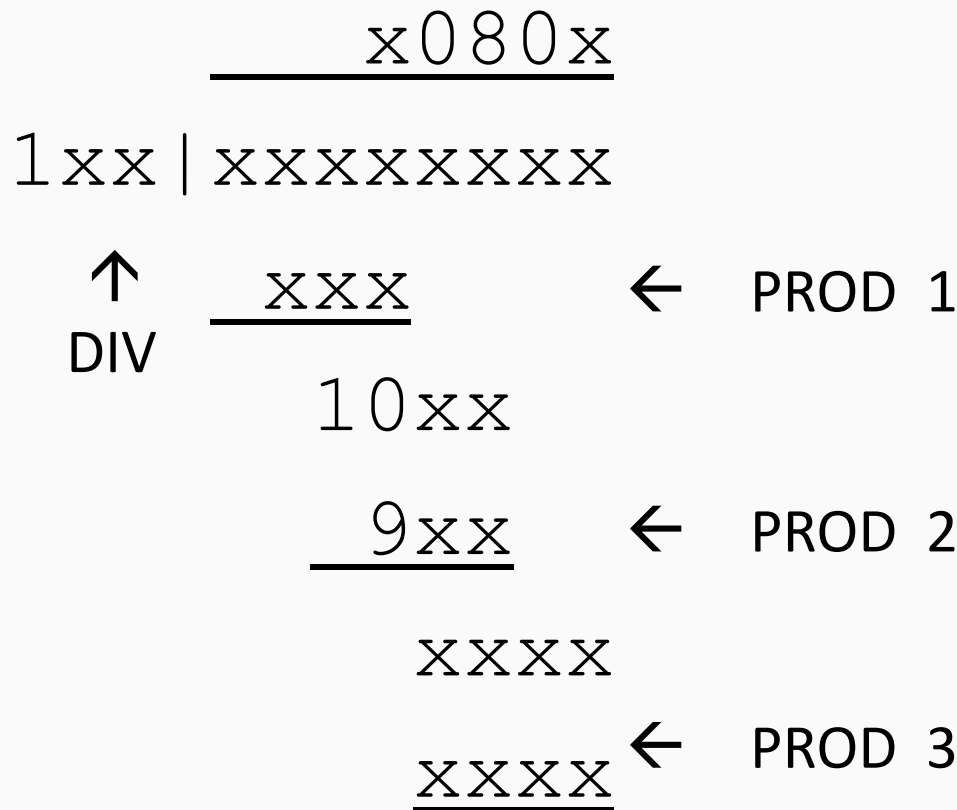
Special features:

PROD 2 is a number between 800-992.

It is subtracted from a 4 place number but yields a 2 place result. The only combination for which this can hold is when a 1 is borrowed to the second column to cancel a 9.

Therefore $DIV * 8 > 900$ so $113 \leq DIV \leq 124$.

10XX
<u>-9XX</u>
XX



PROD 3 is the only product with 4 digits & when subtracted yields 0 so it must be identical to the number it is subtracted from. Which derived from PROD 2.

Multiplying DIV by final quotient digit yields a 4 digit number. Thus the final quotient digit > 8 (which yielded a 3 digit number) therefore it is 9.

Division Problem: Continuing

$\underline{\quad\quad\quad x0809}$

1xx | xxxxxxxxxxx

↑
DIV $\underline{\quad\quad\quad xxx}$ ← PROD 1

 10xx

$\underline{\quad\quad\quad 9xx}$ ← PROD 2

 1xxx

$\underline{\quad\quad\quad 1xxx}$ ← PROD 3

Since $113 \leq \text{DIV} < 124$, PROD 3 must be:
 $9 * 113 = 1017 \leq \text{PROD 3} < 1116 = 9 * 124$

PROD 2

Special features:

10xx

PROD 2 is a number between 800-992.

Only 2 digits must be carried down.

-9xx

What does the one beneath PROD2 imply?

1x

The X's in column have a difference of one. And a one must be borrowed from column 3.

The only pairs for which this holds are 0-9, 0-8, or 1-9 (in the last 2 cases a 1 must be borrowed into the first column).

Thus PROD 2 is either 99X or 98X. Which implies DIV is either 123 or 124.

$$\begin{array}{r}
 \underline{80809} \\
 12x 10 \\
 \leftarrow \text{PROD 1} \\
 \\
 \underline{9xx} \leftarrow \text{PROD 2} \\
 \\
 \leftarrow \text{PROD 3} \\
 \underline{}
 \end{array}$$

Note:

1. Whenever a number is carried down and the result is < DIV we place a 0 in the quotient digit.

2. The PROD 1 subtraction pattern is identical to the PROD 2 subtraction pattern. Thus the first quotient digit must be an 8.

PROD 1

PROD 2

PROD 3

Division Problem: Completion

Problems 10

$$\begin{array}{r} \underline{ 80809} \\ 124 \mid 10020316 \\ \underline{ 992} \\ 1003 \\ \underline{ 992} \\ 1116 \\ \underline{ 1116} \end{array}$$

DIV is either 123 or 124 and the quotient must be 80809. By testing each of these possible divisors by the quotients to produce the products one will find that only 124 yields a result that satisfies all the constraints.