

# CS5014 Fall 2005 Homework Assignment 11

## Solutions

1. [10 points] Navidi Chapter 2, Supplementary Exercise 2.

The probability of needing more than five trials is the probability that the first five all fail. This is  $p(X \neq 6) = \frac{5}{6}$  for each of the first five trials, or a total probability of  $\frac{5^5}{6} = .4019 = 40.2\%$ .

- 2 [10 points] Navidi Section 4.9, Exercise 4.

We need to plot the  $X_i$  (observed values) vs. the computed z-scores for each value, based on its predicted quantile position ( $Q_i$ ). Navidi does it slightly differently, but the graph should look the same.

Value	i	Q(i-0.5)/40	z-score
45	1	0.0125	-2.2414
49	2	0.0375	-1.7805
51	3	0.0625	-1.5341
51	4	0.0875	-1.3563
51	5	0.1125	-1.2133
51	6	0.1375	-1.0916
53	7	0.1625	-0.9842
53	8	0.1875	-0.8871
55	9	0.2125	-0.7978
60	10	0.2375	-0.7144
66	11	0.2625	-0.6357
67	12	0.2875	-0.5607
67	13	0.3125	-0.4888
68	14	0.3375	-0.4193
70	15	0.3625	-0.3518
71	16	0.3875	-0.2858
72	17	0.4125	-0.2211
73	18	0.4375	-0.1573
75	19	0.4625	-0.0941
75	20	0.4875	-0.0313
75	21	0.5125	0.0313
76	22	0.5375	0.0941
76	23	0.5625	0.1573
76	24	0.5875	0.2211

Value	i	Q(i-0.5)/40	z-score
79	25	0.6125	0.2858
79	26	0.6375	0.3518
80	27	0.6625	0.4193
81	28	0.6875	0.4888
82	29	0.7125	0.5607
82	30	0.7375	0.6357
82	31	0.7625	0.7144
83	32	0.7875	0.7978
84	33	0.8125	0.8871
84	34	0.8375	0.9842
85	35	0.8625	1.0916
86	36	0.8875	1.2133
86	37	0.9125	1.3563
86	38	0.9375	1.5341
88	39	0.9625	1.7805
91	40	0.9875	2.2414

Whether this is normal is somewhat in eye of the beholder. But there is a big bulge at the beginning, and at the end it deviates quite a bit from the line. So most people probably won't consider this as coming from a normal distribution.

**3** [10 points] Navidi Chapter 4, Supplementary Exercise 8.

- (a) How much area is under the curve between 9.3 and 10.7? We need to compute the z-scores for  $(10.7 - 10.0)/1.0 = 0.7$  and  $(9.3 - 10.0)/1.0 = -0.7$ . These z-scores are .7580 and 0.2420 respectively, for a difference of .5160 or 51.6%.
- (b) The probability for each resistor being in this range is 51.6%. If we treat each measurement as a Bernoulli trial, we use a binomial distribution  $X \sim Bin(100, 0.5160)$ . However, we can simplify by using the normal approximation  $X \sim N(np, np(1-p)) = N(51.60, 24.97)$ . Thus we solve

$$z = \frac{50 - 51.6}{\sqrt{24.97}} = \frac{-1.6}{4.997} = 0.32.$$

Using the z-table, we get a probability of  $1 - 0.3745 = 62.55$ .

- (c) Solving the inverse problem from part (b), our z-table probability will be  $1 - 0.99 = 0.01$  with a z-value of -2.33. Solve for  $n$  in the equation

$$2.33 = \frac{50 - n(0.516)}{\sqrt{n(0.516)(0.484)}}.$$

This will yield  $n = 122$  for our sample size.

4 [20 points] You may solve this problem analytically or by simulation.

Jack the Gambler has a system. He's going to Las Vegas to try it out. He observes that a roulette wheel lets him win half the time if he bets that the result is an even number, and he receives two times his bet if he wins. So his system works like this: He starts by betting \$1, and he doubles his bet every time he loses. Eventually he must win with a net profit (for that series of bets) of \$1. For example, say he bets \$1 and loses, then \$2 and loses, then \$4 and wins \$8. His net winnings over the series of 3 bets are \$1.

- (a) Assume that the roulette wheel will run 60 times/hour. If Jack plays for two hours (give or take a few minutes to complete the current cycle), how much should he expect to win?

Jack will keep doubling his bet each time until he wins. No matter how many times this takes, his net winning will be \$1. However, the amount of time required to get the \$1 will vary. Half the time, he gets it in one attempt (1 minute), 1/4 of the time he requires 2 attempts (2 min), 1/8 of the time its 3 attempts (3 min) and so on. Thus, we need to know the expected amount of time to win \$1. This will be

$$\sum_{i=0}^{\infty} \frac{i}{2^i} = 2.$$

Thus, the expected cycle time is 2 minutes to win \$1. So he expects to win \$60.

- (b) Unfortunately, Jack has a problem. He's only got \$1000 with him. So if he loses enough times in a row, he's out a lot of money because he won't have enough money left to continue the cycle! If this should happen, he will start over with a new cycle (beginning with a \$1 bet) if he has any money left. With this taken into consideration, how much should he expect to win or lose in 2 hours? In 20 hours?

A major issue is whether Jack gets "wiped out" by losing so many bets in a row that he loses his stake. If he doesn't lose his stake, then he wins \$1 every 2 minutes. But if he loses his stake, he has no hope of making much of it back, so his loss is close to \$1000. To lose his stake, Jack needs to lose either 9 or 10 times in a row. (If he has less

than \$1023 he can only bet 9 times, for a total loss of \$511. If he has gotten up to \$1023, then he can bet a 10th time before losing \$1023.) His chance of a wipeout in 2 hours is relatively low (you can use the binomial distribution to work this out, but it is something under 10%), but then his opportunity to win is also relatively low (again, somewhat less than 10%). When playing for 20 hours, his chance of a wipeout is higher (around 40%), but the amount that he can win is greater (expected winnings with no wipeouts is \$600).

It turns out (according to my simulations) that the amount lost to wipeouts is roughly equal to that gained by the system he is using. Thus, the net expectation is to break even, both at 2 hours and 20 hours.

This might seem surprising if you observe that the expected winnings (with no wipeouts) is a higher percentage of the starting value than the probability of a wipeout. But once a wipeout occurs, the odds of additional wipeouts goes way up because the total amount of money (and thus the number of continuous losses required) goes way down. Once a loser, always a loser!