Multivalued Dependencies & Fourth Normal Form (4NF)

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A New Form of Redundancy

- Multivalued dependencies (MVD's) express a condition among tuples of a relation that exists when the relation is trying to represent more than one many-many relationship.

- Then certain attributes become independent of one another, and their values must appear in all combinations.

Example

Drinkers(name, addr, phones, beersLiked)

- A drinker's phones are independent of the beers they like.
- Thus, each of a drinker's phones appears with each of the beers they like in all combinations.
  - If a drinker has 3 phones and likes 10 beers, then the drinker has 30 tuples
  - where each phone is repeated 10 times and each beer 3 times
- This repetition is unlike redundancy due to FD's, of which name->addr is the only one.

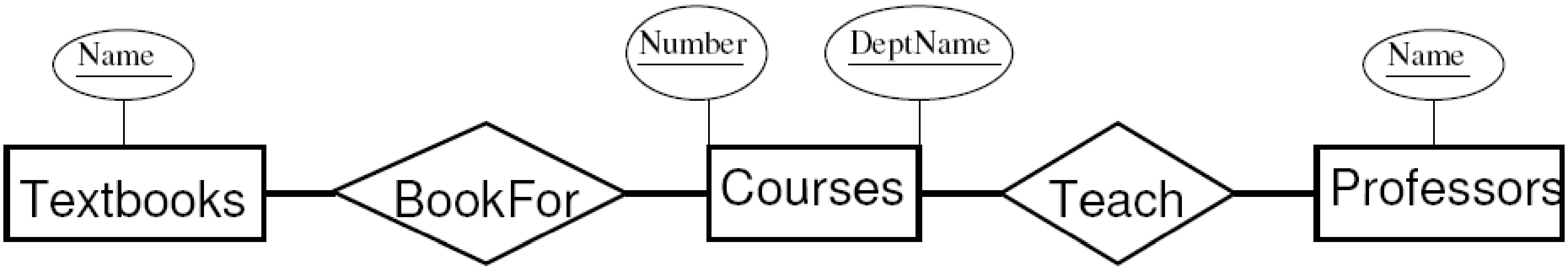
Tuples Implied by Independence

If we have tuples:

name	addr	phones	beersLiked
sue	a	p1	b1
sue	a	p2	b2
sue	a	p2	b1
sue	a	p1	b2

Then these tuples must also be in the relation.

Another Example



- The relation is `Courses(Number, DeptName, Textbook, Professor)`.
  - Each `Course` can have multiple required `Textbooks`.
  - Each `Course` can have multiple `Professors`.
  - `Professors` uses every required textbook while teaching a `Course`.

Number	DeptName	Textbook	Professor
4604	CS	FCDB	Ullman
4604	CS	SQL Made Easy	Ullman
4604	CS	FCDB	Widom
4604	CS	SQL Made Easy	Widom

- The relation is in BCNF since there are no non-trivial FDs.
- Is there any redundancy?

Definition of MVD

- A *multivalued dependency* (MVD) *X* ->->*Y* is an assertion that if two tuples of a relation agree on all the attributes of *X*, then their components in the set of attributes *Y* may be swapped, and the result will be two tuples that are also in the relation.

Definition of MVD

- A multi-valued dependency (MVD or MD) is an assertion that two sets of attributes are independent of each other.
- The *multi-valued dependency* $A_1 A_2 \ldots A_n \twoheadrightarrow B_1 B_2 \ldots B_m$ holds in a relation R if in every instance of R,

  for every pair of tuples t and u in R that agree on all the A's, we can find a tuple v in R that agrees

  1. with both t and u on A's,
  2. with t on the B's, and
  3. with u on all those attributes of R that are not A's or B's.

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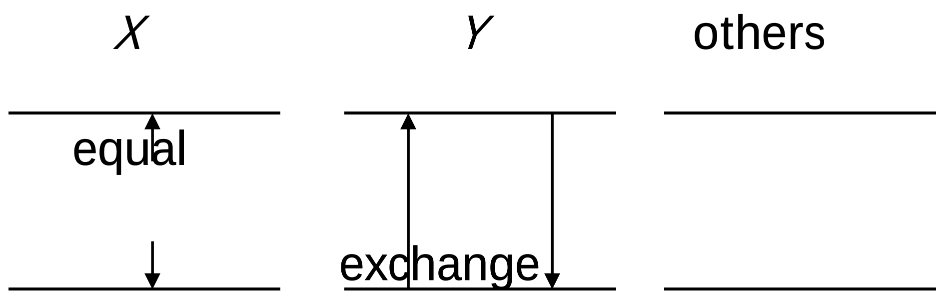
Example

	Number	DeptName	Textbook	Professor
	4604	CS	FCDB	Ullman
t	4604	CS	SQL Made Easy	Ullman
u	4604	CS	FCDB	Widom
v	4604	CS	SQL Made Easy	Widom
	2604	CS	Data Structures	Ullman
	2604	CS	Data Structures	Widom

- Number DeptName $\twoheadrightarrow$ Textbook is an MD. For every pair of tuples t and u that agree on Number and DeptName, we can find a tuple v that agrees
  1. with both t and u on Number and DeptName,
  2. with t on Textbook, and with u on Professor.
- Number DeptName $\twoheadrightarrow$ Professor is an MD. For every pair of tuples t and u that agree on Number and DeptName, we can find a tuple v that agrees
  1. with both t and u on Number and DeptName,
  2. with t on Professor, and with u on Textbook.

Picture of MVD *X* ->->*Y*



Number	DeptName	Textbook	Professor
4604	CS	FCDB	Ullman
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2604	CS	Data Structures	Widom

- Does X -> Y imply X ->> Y ?

MVD Rules

- Every FD is an MVD
  - If *X* ->*Y*, then swapping *Y*'s between two tuples that agree on *X* doesn't change the tuples.
  - Therefore, the "new" tuples are surely in the relation, and we know *X* ->->*Y*.
- Definition of keys depend on FDs and not MDs

Rules for Manipulating MDs

- *Trivial dependencies rule:* If $A \twoheadrightarrow B$ is an MD, then $A \twoheadrightarrow AB$ is also an MD.

Splitting Doesn’t Hold

- Like FD’s, we cannot generally split the left side of an MVD.

- But unlike FD’s, we cannot split the right side either --- sometimes you have to leave several attributes on the right side.

Another Example

- Consider a drinkers relation:

Drinkers(name, areaCode, phone, beersLiked, manf)

- A drinker can have several phones, with the number divided between areaCode and phone (last 7 digits).
- A drinker can like several beers, each with its own manufacturer.

Example, Continued

- Since the areaCode-phone combinations for a drinker are independent of the beersLiked-manf combinations, we expect that the following MVD's hold:

    name ->-> areaCode phone

    name ->-> beersLiked manf

Example Data

Here is possible data satisfying these MVD's:

name	areaCode	phone	beersLiked	manf
Sue	650	555-1111	Bud	A.B.
Sue	650	555-1111	WickedAle	Pete's
Sue	415	555-9999	Bud	A.B.
Sue	415	555-9999	WickedAle	Pete's

But we cannot swap area codes or phones my themselves.
That is, neither name ->-> areaCode nor name ->-> phone
holds for this relation.

Fourth Normal Form

- The redundancy that comes from MVD's is not removable by putting the database schema in BCNF.

- There is a stronger normal form, called 4NF, that (intuitively) treats MVD's as FD's when it comes to decomposition, but not when determining keys of the relation.

4NF Definition

- A relation *R* is in 4NF if whenever *X* ->->*Y* is a nontrivial MVD, then *X* is a superkey.
  - Nontrivial means that:
    1. *Y* is not a subset of *X*, and
    2. *X* and *Y* are not, together, all the attributes.
  - Note that the definition of "superkey" still depends on FD's only.

BCNF Versus 4NF

- Remember that every FD *X* ->*Y* is also an MVD, *X* ->->*Y*.
- Thus, if *R* is in 4NF, it is certainly in BCNF.
  - Because any BCNF violation is a 4NF violation.

- But *R* could be in BCNF and not 4NF, because MVD's are "invisible" to BCNF.

Decomposition and 4NF

- If X ->->Y is a 4NF violation for relation R, we can decompose R using the same technique as for BCNF.

1. XY is one of the decomposed relations.
2. All but $Y – X$ is the other.

Example

Drinkers(name, addr, phones, beersLiked)

FD: name -> addr

MVD's: name ->-> phones

name ->-> beersLiked

- Key is
  - {name, phones, beersLiked}.
- Which dependencies violate 4NF ?
  - All

Example, Continued

- Decompose using name -> addr:

1. Drinkers1(name, addr)
   - In 4NF, only dependency is name -> addr.
2. Drinkers2(name, phones, beersLiked)
   - Not in 4NF. MVD's name ->-> phones and name ->-> beersLiked apply.
   - Key ?
     - No FDs, so all three attributes form the key.

Example: Decompose Drinkers2

- Either MVD name ->-> phones or name ->-> beersLiked tells us to decompose to:
  - Drinkers3(name, phones)
  - Drinkers4(name, beersLiked)

Relationships Among Normal Forms

- 4NF implies BCNF, i.e., if a relation is in 4NF, it is also in BCNF.
- BCNF implies 3NF, i.e., if a relation is in BCNF, it is also in 3NF.

Property	3NF	BCNF	4NF
Eliminates redundancy due to FDs	Maybe	Yes	Yes
Eliminates redundancy due to MDs			
Preserves FDs			
Preserves MDs			

kNFs

- First Normal Form: each attribute is atomic.